(i) CAPACITORS

4.1 INTRODUCTION

Capacitance of a conductor. If a charge q is given to an isolated conductor, its voltage is increased by an amount V. For a given conductor, the ratio Q/V is independent of Q and depends only on the size and shape of the conductor. The ratio Q/V is called the *capacitance of the conductor*, and is denoted by C.

$\therefore C = Q/V$. Hence *the capacitance of a conductor is defined as the ratio of the charge given to the increase in the potential of the conductor.*

The capacitance of a conductor is also defined as the amount of charge that should be given to it to increase its potential by unity.

The unit of capacitance is *farad. A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it raises its potential by 1 volt.* $1 \mu F = 10^{-6} F$; $1 pF = 10^{-12} F$.

Principle of a Capacitor. Suppose an insulated metallic plate A is given a positive charge Q and its potential is V (Fig. 4.1*a*). Its capacitance $C = Q/V$. Let another insulated metal plate B be brought near A. Negative charge is induced on that side of B which is nearer to A. An equal positive charge is induced on the other side of B. The negative charge on B decreases the potential of A. The positive charge on B increases the potential of A. But the negative charge on B is nearer to A than the positive charge on B. So the net effect is that the potential of A decreases. Thus the capacitance of A is increased.

The positive charge on B is neutralized by connecting the back side of B to earth (Fig. 4.1*b*). Then the potential of A decreases still further. Thus the capacitance of A is considerably increased.

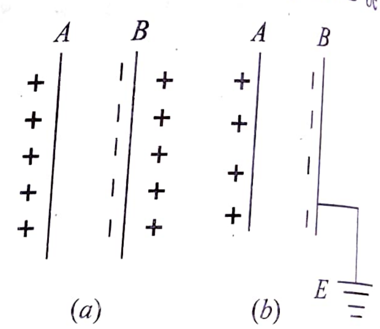

FIG. 4.1

A capacitor in general consists of two conductors one positively charged and the other earthed. The conductors are called *plates*. The capacitance depends on the geometry of the conductors and the permittivity of the medium separating them. A capacitor is a device for storing charge.

4.2 CAPACITANCE OF A SPHERICAL CAPACITOR (OUTER SPHERE EARTHED)

Let A and B be two concentric metal spheres of radii a and b respectively with air as the intervening medium (Fig. 4.2). The outer sphere B is earthed. A charge $+q$ is given to the inner sphere. The induced charge on the inner surface of the outer sphere is $-q$. P is a point at a distance r from the common centre O.

Electric field at $P = \mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r^2}\right)\hat{r}$ ...(1)

where $\hat{r}$ is the unit vector along $\overrightarrow{OP}$.

The potential difference between the spheres A and B is given by

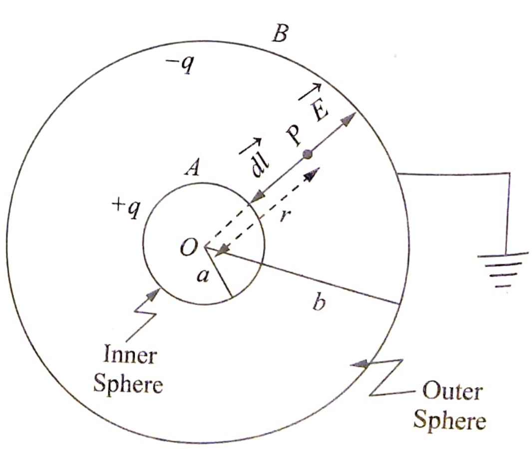

FIG. 4.2

$$V = -\int_b^a \mathbf{E} \cdot d\mathbf{l} \qquad ...(2)$$

Here, $d\mathbf{l}$ is the differential vector displacement along a path *from B* to *A*.

But $\mathbf{E} \cdot d\mathbf{l} = E\,dl \cos 180° = -E\,dl.$

Further, in moving a distance dl in the direction of motion, we are moving in the direction of r *decreasing*, so that $dl = -dr$. Hence,

$$\mathbf{E} \cdot d\mathbf{l} = E\, dr.$$

Eq. (2) becomes $V = -\int_b^a E\, dr$.

Putting the value of E from Eq. (1), we get

$$V = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0}\left\{-\frac{1}{r}\right\}_b^a$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{a}-\frac{1}{b}\right\} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{b-a}{ab}$$

∴ Capacitance of the spherical capacitor

$$C = \frac{q}{V} = \frac{q}{\left(\frac{q}{4\pi\varepsilon_0}\right)\left(\frac{b-a}{ab}\right)} = 4\pi\varepsilon_0\frac{ab}{(b-a)} \quad ...(3)$$

Note. Eq. (3) can be written in the form

$$C = 4\pi\varepsilon_0\frac{ab}{b-a} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

When $b \to \infty$, $C = 4\pi\varepsilon_0 a$.

This is the *capacitance of an isolated conducting sphere* of radius a.

4.3 CAPACITANCE OF A SPHERICAL CAPACITOR (INNER SPHERE EARTHED)

A and *B* are two spheres of radii a and b (Fig. 4.3). Suppose a charge $+q$ is given to the outer sphere *B*. $+q$ is distributed on its inner and outer surfaces by amounts $+q_1$ and $+q_2$ respectively, so that $q = q_1 + q_2$. The charge $+q_1$ on the inner surface of *B* induces a charge $-q_1$ (bound charge) on the outer surface of *A* and charge $+q_1$ on the inner surface of *A*. The charge $+q_1$ on the inner surface of *A*, being free, leaks to the earth.

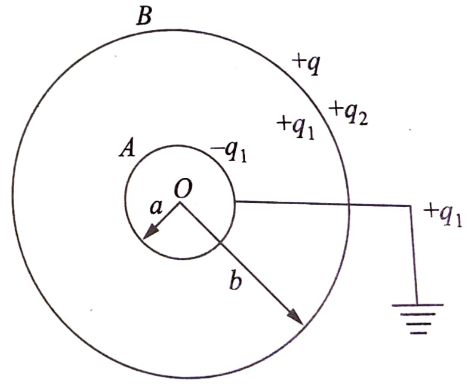

FIG. 4.3

The two spheres now behave as two capacitors connected in parallel.

(*i*) The inner sphere of radius a and the inner surface of outer sphere form a capacitor of capacitance

$$C_1 = \frac{4\pi\varepsilon_0 ab}{b-a} \quad \text{(if the dielectric is air)}$$

(*ii*) The outer surface of *B* and the earth form a capacitor of capacitance

$$C_2 = 4\pi\varepsilon_0 b.$$

Total capacitance

$$C = C_1 + C_2 = \frac{4\pi\varepsilon_0 ab}{(b-a)} + 4\pi\varepsilon_0 b$$

∴

$$C = \frac{4\pi\varepsilon_0 b^2}{b-a}$$

4.4 CAPACITANCE OF A CYLINDRICAL CAPACITOR

Consider a cylindrical capacitor formed by two coaxial cylinders A and B of radii a and b respectively and each of length l. Air is the medium between A and B. The outer cylinder B is earthed (Fig. 4.4 a).

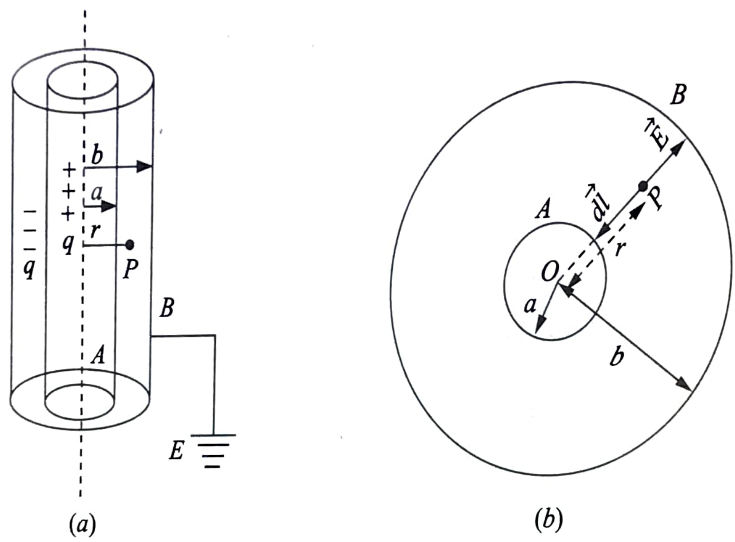

FIG. 4.4

If a charge $+q$ is given to the inner cylinder, then an equal charge $-q$ is induced on the inner surface of the outer cylinder and a charge $+q$ on the outer surface of the outer cylinder. The charge $+q$ induced on the outer surface of the outer cylinder flows to the earth.

The electric field at a point P in the space between the two cylinders at a distance r from the axis is

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}. \qquad \text{...(1)}$$

The potential difference V between the cylinders A and B is

$$V = -\int_b^a \mathbf{E}\cdot d\mathbf{l} \qquad \text{... (2)}$$

Here, $d\mathbf{l}$ is the vector displacement along a path from B to A (Fig. 4.4 b).

Now, $\mathbf{E}$ is radially outward and $d\mathbf{l}$ is inward. Therefore

$$\mathbf{E}\,.\,d\mathbf{l} = E\,dl\cos 180° = -E\,dl.$$

As we move a distance dl from B to A, we move in the direction of decreasing r. So $dl = -dr$. Thus

$$E\cdot d\mathbf{l} = Edr.$$

Eq. (2) becomes,

$$V = -\int_b^a E\,dr$$

$$= -\frac{q}{2\pi\varepsilon_0 l}\int_b^a \frac{dr}{r} \qquad \text{[From Eq. (1)]}$$

$$= -\frac{q}{2\pi\varepsilon_0 l}\{\log_e r\}_b^a = -\frac{q}{2\pi\varepsilon_0 l}\{\log_e a - \log_e b\}$$

$$= \frac{q}{2\pi\varepsilon_0 l}\log_e\frac{b}{a}.$$

Hence the capacitance of the cylindrical capacitor is

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\log_e(b/a)}$$

Examples of practical Cylindrical Capacitors

(i) The *co-axial cable* consists of a cylindrical metal shield, a co-axial central conductor and an interposed dielectric.

(ii) A *submarine cable* consists of strands of copper separated from the surrounding water by a suitable insulating cable. It thus acts as a cylindrical capacitor. The copper strands form the inner cylinder. The surrounding water acts as the outer cylinder. The insulating casing acts as the dielectric.

4.5 CAPACITANCE OF A PARALLEL PLATE CAPACITOR

The parallel plate capacitor consists of two parallel metal plates each of area A and separated by a distance d (Fig. 4.5). The medium between the plates is air. A charge $+q$ is given to the plate P. It induces a charge $-q$ on the upper surface of the earthed plate Q. d is kept small compared with the plate dimensions to enable us to ignore the fringing effects near the ends.

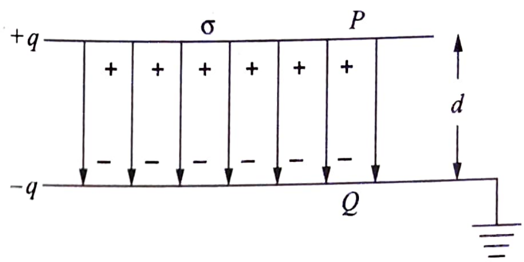

FIG. 4.5

Thus electric lines of force starting from plate P and ending at the plate Q are parallel to each other and perpendicular to the plates.

By the application of Gauss's law,

$$\left.\begin{array}{r}\text{electric field at a point}\\ \text{between the two plates}\end{array}\right\} = E = \frac{\sigma}{\varepsilon_0}$$

Here, σ = surface density of charge = q/A.

Potential difference between the plates P and Q is

$$V = \int_d^0 -E\,dr = \int_d^0 -\frac{\sigma}{\varepsilon_0}dr = \frac{\sigma d}{\varepsilon_0}$$

The capacitance of the parallel plate capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{(\sigma d/\varepsilon_0)} = \frac{\varepsilon_0 A}{d}$$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d}$$

4.5.1 Effect of a Dielectric

Relations obtained earlier for the capacitances hold only when the plates are in vacuum or air. In actual capacitors, the region between its two conductors is filled with an insulator (or *dielectric*) say mica or oil. Faraday found that the capacitance of a capacitor increases if a dielectric is placed between the plates.

If C is the capacitance of a capacitor with vacuum and C' is its capacitance with dielectric, then the ratio $C'/C = \varepsilon_r$ is called the *relative permittivity of the medium.*

$$\therefore \quad C' = \varepsilon_r C$$

The capacitance of a capacitor with a medium of relative permittivity ε_r between its two conductors is ε_r times the value given by the above formulae. Therefore

$$C_d = \frac{4\pi\varepsilon_r\varepsilon_0 ab}{(b-a)} \quad \text{for a spherical capacitor (outer sphere earthed)}$$

$$= \frac{2\pi\varepsilon_0\varepsilon_r l}{\log_e(b/a)} \quad \text{for cylindrical capacitor}$$

$$= \frac{\varepsilon_r\varepsilon_0 A}{d} \quad \text{for parallel plate capacitor}$$

4.5.2 Capacitance of a Parallel Plate Capacitor Partly Filled with a Dielectric Slab

P and Q are the conducting plates of a parallel plate capacitor, each of area A placed at a distance d apart. Suppose a dielectric slab of thickness t and relative permittivity ε_r is introduced between the plates (Fig. 4.6). P is given a positive charge so that the surface charge density on it is σ.

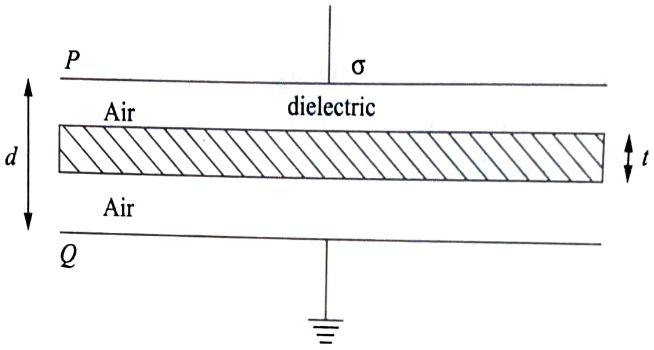

FIG. 4.6

Thickness of dielectric slab $= t$

Thickness of air portion $= (d - t)$

Electric field at any point in the air space between the plates $= E = \dfrac{\sigma}{\varepsilon_0}$

Electric field at any point in the dielectric slab $= E' = \dfrac{\sigma}{\varepsilon_r\varepsilon_0}$

The potential difference V between the plates is the work done in carrying a unit positive charge from one plate to other in the field E over a length $(d - t)$ and in the field E' over a length t. Thus

$$V = E(d-t) + E't = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma t}{\varepsilon_r\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

The charge on the plate $P = q = \sigma A$.

Hence the capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{\sigma A}{\dfrac{\sigma}{\varepsilon_0}\left[(d-t) + \dfrac{t}{\varepsilon_r}\right]} = \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{\varepsilon_r}}$$

Example 1. *The radii of the inner and outer spheres of a spherical capacitor are 4×10^{-2} metre and 6×10^{-2} metre. If the dielectric medium between the plates is air, calculate the capacitance of the spherical capacitor if the outer sphere is earthed and the inner sphere is positively charged.*

Solution. $C = \dfrac{4\pi\varepsilon_0 \times ab}{b-a} = \dfrac{1}{9 \times 10^9} \times \dfrac{4 \times 10^{-2} \times 6 \times 10^{-2}}{2 \times 10^{-2}}$

$= 1.33 \times 10^{-11}$ farad.

Example 2. *A sphere of 10 cm diameter is suspended within a hollow sphere of 12 cm diameter. If the inner sphere be charged to a potential of 15000 volt and the outer sphere be earthed, find the charge on the inner sphere.*

Solution. $C = \frac{4\pi\varepsilon_0 ab}{b-a} = \left(\frac{1}{9\times10^9}\right)\frac{0.05\times0.06}{0.01} = 3.333\times10^{-11}\ F$

$\therefore$ Charge on the inner sphere $= q = CV = (3.333\times10^{-11})\times15000 = 5\times10^{-7}\ C$

Example 3. *Calculate the capacitance of a sphere of 20 cm diameter inside which there is an earth connected sphere of 10 cm diameter, the medium between the spheres being air.*

Solution. $C = \frac{4\pi\varepsilon_0 b^2}{b-a} = \left(\frac{1}{9\times10^9}\right)\frac{(0.1)^2}{(0.1-0.05)} = 2.22\times10^{-11}\ F.$

Example 4. *Calculate the capacitance of a cylindrical capacitor if the radii of the inner and outer cylinders are 2 × 10^{-3} metre and 8 × 10^{-3} metre, the outer cylinder being earthed and the inner cylinder is given a positive charge. The relative permittivity of the dielectric medium between the cylinders is 5 and the length of the cylinders is 6 metre.*

Solution. $C = \frac{2\pi\varepsilon_r\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)} = \frac{2\times3.1416\times5\times8.854\times10^{-12}\times6}{2.303\times\log_{10}\left(\frac{0.008}{0.002}\right)}$

$= 1.203\times10^{-9}\ F.$

Example 5. *A cable has a wire of radius 1 mm and it is surrounded by a thin metallic sheet of radius 6 mm. The space between the cable and the sheet is filled with a material of dielectric constant 2.05. What is the capacitance of 8 km length cable ?*

Solution. Capacitance of cylindrical capacitor

$$C = \frac{2\pi\varepsilon_r\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)} = \frac{2\pi\times2.05\times(8.854\times10^{-12})\times8000}{2.3026\times\log_{10}\left(\frac{0.006}{0.001}\right)}$$

$= 0.5092\times10^{-6}\ F.$

Example 6. *The area of each plate of a parallel plate capacitor is 4 × 10^{-2} square metre. If the thickness of the dielectric medium between the plates is 10^{-3} metre and the relative permittivity of the dielectric is 7, find the capacitance of the capacitor.*

Solution. $C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{7\times8.854\times10^{-12}\times4\times10^{-2}}{10^{-3}} = 2.478\times10^{-9}\ F$

Example 7. *A capacitor is made up of two plates separated by a sheet of insulating material 3mm thick and of relative permittivity (dielectric constant) 4. The distance between the plates is increased to allow the insertion of a second sheet 5 mm thick and relative permittivity ε_r. If the capacitance of the capacitor so formed is one half of the original capacitance, find the value of ε_r.*

Solution. Initial capacitance of the capacitor is

$$C = \frac{\varepsilon_{r_1}\varepsilon_0 A}{d} = \frac{4\varepsilon_0 A}{3\times10^{-3}} \quad (\because \varepsilon_{r_1} = 4, d = 3\times10^{-3}\ \text{m}) \qquad ...(1)$$

After the insertion of the second sheet, the interspace is filled with two dielectrics of thickness $t_1 = 3$ mm, $\varepsilon_{r_1} = 4$ and $t_2 = 5$ mm and unknown relative permittivity ε_r.
The new capacitance is

$$\therefore \qquad C' = \frac{\varepsilon_0 A}{\dfrac{t_1}{\varepsilon_{r_1}} + \dfrac{t_2}{\varepsilon_r}} = \frac{\varepsilon_0 A}{\dfrac{3\times10^{-3}}{4} + \dfrac{5\times10^{-3}}{\varepsilon_r}} \qquad ...(2)$$

$$\frac{C}{C'} = \frac{4}{(3\times10^{-3})} \times \left(\frac{3\times10^{-3}}{4} + \frac{5\times10^{-3}}{\varepsilon_r}\right) = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{\varepsilon_r}\right)$$

It is given that

$$C' = \frac{1}{2}C$$

$$\therefore \qquad 2 = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{\varepsilon_r}\right)$$

Simplifying

$$\varepsilon_r = 6.67$$

Example 8. *A parallel plate capacitor of plate area 10^{-2} m^2 and plate separation 10^{-2} m is charged to 100 volts. Then, after removing the charging battery, a slab of insulating material of thickness 0.5×10^{-2} m and relative permittivity 7 is inserted between the plates. Calculate the free charge on the plates of the capacitor, electric field intensity in air, electric field intensity in the dielectric, potential difference between the plates and the capacitance (with dielectric present).*

Solution. The capacitance C_0, before the slab is inserted, is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.9\times10^{-12})\times(10^{-2})}{10^{-2}} = 8.9\times10^{-12}\ F.$$

Therefore, the free charge is

$$q = C_0 V_0 = (8.9\times10^{-12})\times 100 = \mathbf{8.9\times10^{-10}}\ C$$

The electric field intensity in air is

$$E_0 = \frac{V_0}{d} = \frac{100}{10^{-2}} = \mathbf{1.0\times10^{4}}\ Vm^{-1}$$

The electric field intensity in the dielectric is

$$E = \frac{E_0}{\varepsilon_r} = \frac{1.0\times10^4}{7} = \mathbf{1.43\times10^{3}}\ Vm^{-1}$$

The potential difference between the plates with dielectric present is

$$V = E_0(d-t) + Et$$

$$= (1.0\times10^4)(10^{-2} - 0.5\times10^{-2}) + (1.43\times10^3)(0.5\times10^{-2})$$

$$= \mathbf{57\ volt.}$$

The (free) charge on the plates is same as before.
The capacitance (with dielectric present) is

$$C = \frac{q}{V} = \frac{8.9\times10^{-10}\,\text{coul.}}{57\ \text{volt}} = 16\times10^{-12}\ F = 16pF$$

Example 9. *It is required to construct a parallel plate capacitor of capacitance 0.5 microfarad using paper sheets of thickness 4×10^{-5} metre as dielectric. How many circular metal foils of radius 2×10^{-1} metre are required given that the relative permittivity of paper is 5 and $\varepsilon_0 = 8.9 \times 10^{-12}$ farad per metre.*

Solution. Let the number of metal foils required be n.

They will form $(n-1)$ capacitors.

The capacitance of the arrangement is given by

$$C = \frac{\varepsilon_r \varepsilon_0 (n-1) A}{d}$$

$$0.5 \times 10^{-6} = \frac{5 \times 8.9 \times 10^{-12} (n-1) \times 3.1416 \times 0.2 \times 0.2}{4 \times 10^{-5}}$$

$$n - 1 = 18$$

$$\therefore \quad n = 19.$$

4.6 CAPACITORS IN SERIES AND PARALLEL

(i) Capacitors in Series. Let C_1, C_2, C_3 *be the capacitances of* three capacitors connected in series (Fig. 4.7). Let V be the potential difference applied across the series combination. Here each capacitor carries the same amount of charge q. Let V_1, V_2, V_3 be the potential difference across the capacitors C_1, C_2, C_3 respectively. Thus

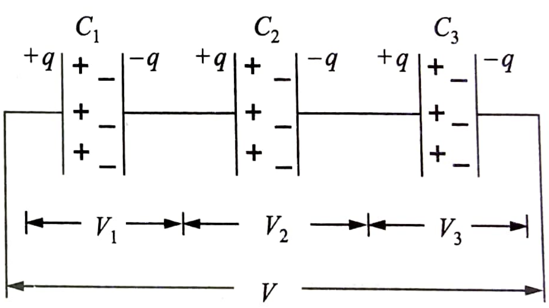

FIG. 4.7

$$V = V_1 + V_2 + V_3$$

But, $$V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2} \text{ and } V_3 = \frac{q}{C_3}$$

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{...(1)}$$

If C_s be the effective capacitance, it should acquire a charge q when a voltage V is applied across it. Hence

$$V = q/C_S \quad \text{...(2)}$$

From (1) and (2), $$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{...(3)}$$

(ii) Capacitors in parallel. Fig. 4.8 shows three capacitors of capacitances C_1, C_2, C_3 connected in parallel. Let the terminals A and B be connected to a potential difference V. The potential difference across each capacitor is the same. The charges on the three capacitors are respectively,

$$q_1 = C_1 V, q_2 = C_2 V, q_3 = C_3 V.$$

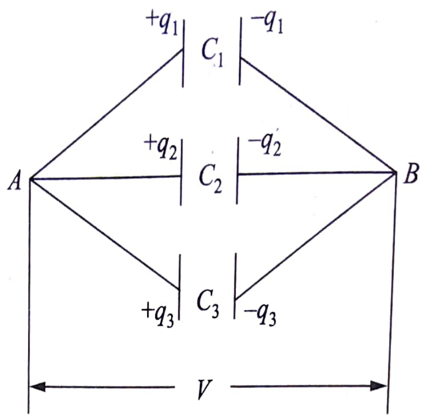

FIG. 4.8

The total charge on the system of capacitors is

$$q = q_1 + q_2 + q_3$$
$$= C_1V + C_2V + C_3V$$
$$= V(C_1 + C_2 + C_3).$$

But $q = C_pV$, where C_p is the equivalent capacitance of the system.

$\therefore$ $$C_PV = V(C_1 + C_2 + C_3)$$

or $$C_P = C_1 + C_2 + C_3$$

Example. *The area of each plate of a parallel plate capacitor is A, and their separation is d. It is filled with two dielectrics of dielectric constants K_1 and K_2. Calculate the capacitance when the dielectrics are filled as in Fig. 4.9 (a), as in (b).*

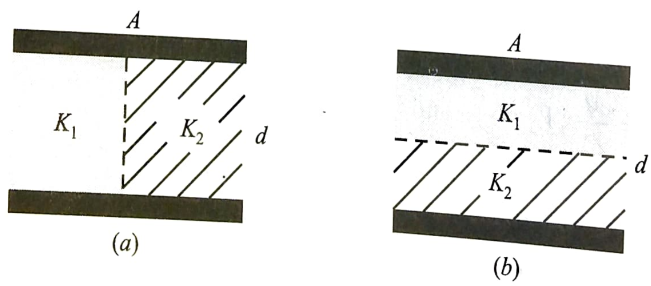

FIG. 4.9

Solution. (*a*) The given arrangement can be considered as a combination of two capacitors in parallel, each with plate area $A/2$ and separation d. Thus,

$$C = C_1 + C_2.$$

Now,

$$C_1 = \frac{K_1\varepsilon_0(A/2)}{d} \text{ and } C_2 = \frac{K_2\varepsilon_0(A/2)}{d}.$$

$$C = \frac{\varepsilon_0 A}{2d}(K_1 + K_2).$$

(*b*) The given arrangement can be considered as a combination of two capacitors in series, each with plate area A and separation $d/2$. Thus

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C = \frac{C_1C_2}{C_1 + C_2}.$$

Here
$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} \text{ and } C_2 = \frac{K_2 \varepsilon_0 A}{d/2}.$$

$\therefore$
$$C = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

4.7 PARALLEL PLATE CAPACITOR FILLED WITH A DIELECTRIC OF LINEARLY INCREASING DIELECTRIC CONSTANT

The space between the plates of a parallel-plate capacitor is filled with a dielectric whose dielectric constant increases linearly from one plate to the other. d is the distance between the two plates (Fig. 4.10). K_1 and K_2 are the values of the dielectric constant at the two plates.

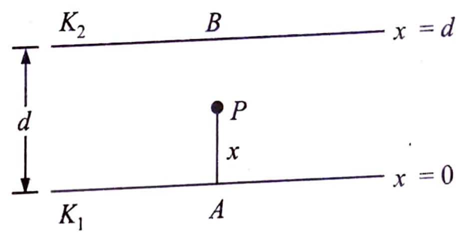

FIG. 4.10

Let us calculate the capacitance per unit area of the capacitor.

Let K_x be the dielectric constant at point P at a distance x from the plate A of the capacitor.

The dielectric constant increases uniformly from one plate to the other. So we can write

$$K_x = K_1 (1 + \alpha x)$$

- α is a constant for the dielectric.

α denotes the rate of change of K along x direction.

Field at P is given by $E_x = \dfrac{\sigma}{\epsilon_0 K_x} = \dfrac{\sigma}{\epsilon_0 K_1 (1 + \alpha x)}$

Here, σ is the surface charge density on the plates.

or
$$\frac{dV}{dx} = \frac{\sigma}{\epsilon_0 K_1 (1 + \alpha x)}$$

or
$$dV = \frac{\sigma dx}{\epsilon_0 K_1 (1 + \alpha x)} = \frac{\sigma}{K_1 \epsilon_0 \alpha}\left[\frac{\alpha dx}{1 + \alpha x}\right] = \frac{\sigma}{K_1 \epsilon_0 \alpha}\left[\frac{d(1 + \alpha x)}{(1 + \alpha x)}\right]$$

Thus the P.D. between the plates is

$$V = \int dV = \frac{\sigma}{K_1 \epsilon_0 \alpha}\int_{x=0}^{d} \frac{d(1 + \alpha x)}{(1 + \alpha x)} = \frac{\sigma}{K_1 \epsilon_0 \alpha} \log_e (1 + \alpha d)$$

$\therefore$ The capacitance per unit area of the capacitor is

$$= \frac{\text{charge/unit area}}{\text{P.D.}} = \frac{\sigma}{V} = \frac{K_1 \epsilon_0 \alpha}{\log_e (1 + \alpha d)}$$

But $K_2 = K_1 (1 + \alpha d)$ $\quad\therefore\quad$ $\alpha = \dfrac{(K_2 - K_1)}{K_1 d}$

$\therefore$
$$\text{Capacitance/unit area} = \frac{\epsilon_0 (K_2 - K_1)}{d \log_e \left(\dfrac{K_2}{K_1}\right)}$$

4.8 PRINCIPLE OF THE VARIABLE AIR CAPACITOR

A capacitor is composed of a pile of n plates of alternate polarity, each having an area A, separated by thin layers of dielectric of thickness d and dielectric constant K. Let us find its capacitance. Large standard capacitors consist of a large number of parallel plate capacitors in parallel. Alternate plates are connected together and brought out to two insulated terminals T_1 and T_2 (Fig. 4.11).

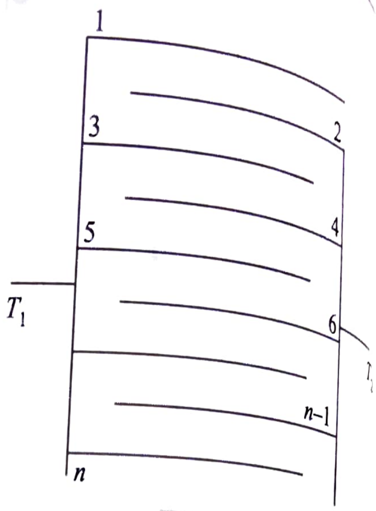

FIG. 4.11

1, 3, 5, ..., n, plates are earthed. The plates 2, 4, 6, ..., $(n-1)$ are connected to a potential V.

The system is equivalent to $(n-1)$ parallel-plate capacitors, connected in parallel, each of which has a capacitance $= \epsilon_0 KA/d$.

The capacitnace of the system is, therefore, given by

$$C = (n-1)\frac{\epsilon_0 KA}{d}$$

The variable air capacitor or tuning capacitor is based on this principle.

4.9 ENERGY STORED IN A CHARGED CAPACITOR

Let q' be the charge and V' the potential difference established between the plates of the capacitor at any instant during the process of charging. If an additional charge dq' is given to the plates, the work done by the battery is given by

$$dW = V'dq' = \left(\frac{q'}{C}\right)dq' \qquad \left(\because \; V' = \frac{q'}{C}\right)$$

Total work done to charge a capacitor to a charge q is

$$W = \int dW = \int_0^q \frac{q'}{C}dq' = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electrostatic potential energy in the capacitor.

$$\therefore \quad U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad (\because \; q = CV)$$

This energy can be recovered if the capacitor is allowed to discharge.

Energy Density. Consider a parallel plate capacitor of area A and plate separation d.

Energy of the capacitor $= U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2$

Volume of the space between the plates $= Ad$.

Energy density u is the potential energy per unit volume.

$$\therefore \quad u = \frac{U}{Ad} = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}V^2\right) \times \frac{1}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

$$\therefore \quad u = \frac{1}{2}\varepsilon_0 E^2 \qquad (\because \; V/d = E)$$

Thus we can associate an electrostatic energy density $u = \frac{1}{2}\varepsilon_0 E^2$ with every point in space where an electric field $\mathbf{E}$ exists.

Example 1. *A co-axial cable consists of a copper core of 1 mm radius within an outer metal sheath of 1 cm, radius separated by an insulating material of dielectric constant 5. What is the capacitance per metre length of the cable in pico-farad ?*

How much energy is stored in 10 km length of this cable when 10,000 volts is applied between the core and the sheath ?

Solution. The capacitance per metre length of the cable is given by

$$C = \frac{2\pi\varepsilon_r\varepsilon_0}{2.3026 \times \log_{10}\left(\frac{b}{a}\right)} = \frac{2 \times 3.14 \times 5 \times (8.85 \times 10^{-12})}{2.3026 \log_{10}\left(\frac{.01}{.001}\right)}$$

$$= \frac{2 \times 3.14 \times 5 \times 8.85 \times 10^{-12}}{2.3026 \times 1} = 120.7 \times 10^{-12} F$$

The capacitance of 10^4 m length of the cable is

$$C' = (120.7 \times 10^{-12}) \times 10^4 = 1.207 \times 10^{-6} F.$$

Energy stored in 10 km length of the cable is

$$U = \frac{1}{2}C'V^2 = \frac{1}{2}(1.207 \times 10^{-6})(10000)^2 = 60.35 \text{ J}.$$

Example 2. *The capacitance of a parallel plate capacitor is 400 pico farad and its plates are separated by 2 mm of air. (i) What will be the energy when it is charged to 1500 volts? (ii) What will be the p.d. with same charge if plate separation is doubled? (iii) How much energy is needed to double the distance between its plates ?*

Solution. Here, $C = 400 \times 10^{-12}$ F, $d = 2 \times 10^{-3}$ m, $V = 1500$ volts.

(*i*) Energy of the capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}(400 \times 10^{-12})(1500)^2 = 4.5 \times 10^{-4} \text{ J}.$$

(*ii*) Charge on the capacitor

$$q = CV = (400 \times 10^{-12}) \times 1500 = 6 \times 10^{-7} C.$$

Capacitance of parallel plate capacitor $C = \varepsilon_0 A/d$
If d is doubled, the capacitance C' is halved.

$$\therefore \quad C' = 200 \times 10^{-12} F.$$

For the same charge q, let V' be the new potential difference.

$$V' = \frac{q}{C'} = \frac{6 \times 10^{-7}}{200 \times 10^{-12}} = 3000 \text{ volts}.$$

(*iii*) The energy required to double the distance between the plates

$$= \text{Final energy} - \text{Initial energy} = \frac{1}{2}C'V'^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 200 \times 10^{-12} \times (3000)^2 - 4.5 \times 10^{-4}$$

$$= 9 \times 10^{-4} - 4.5 \times 10^{-4} = \mathbf{4.5 \times 10^{-4}\ J.}$$

Example 3. *An isolated metal sphere whose diameter is 10 cm has a potential difference of 8000 volts. What is the energy density at the surface of the sphere ?*

Solution. Energy density $= u = \frac{1}{2}\varepsilon_0 E^2$

For a spherical conductor of radius R, $C = 4\pi\varepsilon_0 R$
Charge on the sphere $= q = CV = 4\pi\varepsilon_0 RV$
Electric field on the surface of the metal sphere is

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 R^2} = \frac{V}{R}$$

Here, $V = 8000$ volts ; $R = 0.05$ m.

$$\therefore \quad u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{V^2}{R^2}$$

$$= \frac{1}{2} \times (8.85 \times 10^{-12}) \frac{(8000)^2}{(0.05)^2} = 0.1133 \text{ Jm}^{-3}$$

4.10 CHANGE IN ENERGY OF A PARALLEL PLATE CAPACITOR ON THE INTRODUCTION OF A SLAB OF RELATIVE PERMITTIVITY ε_r BETWEEN THE PLATES

Case (*i*). When the charge remains the same

e.g., when the capacitor is charged by connecting the plates to the + ve and – ve ends of a battery, and the battery is then withdrawn.

(*a*) Without the slab, energy stored

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\varepsilon_0 A} \quad \text{...(1)}$$

(*b*) Let a slab of thickness d and relative permittivity ε_r, which fills the space between the plates, be introduced. Now,

Capacitance $C' = \varepsilon_0 \varepsilon_r A/d$.

With the slab, the energy stored is

$$U' = \frac{q^2}{2C'} = \frac{q^2 d}{2\varepsilon_0 \varepsilon_r A} \quad \text{...(2)}$$

Thus the energy will *decrease* by a factor $1/\varepsilon_r$.

When a slab of thickness t, $(t < d)$, is introduced,

$$C' = \frac{\varepsilon_0 A}{(d - t) + (t/\varepsilon_r)}$$

$$U' = \frac{q^2}{2C'} = \frac{q^2[(d - t) + (t/\varepsilon_r)]}{2\varepsilon_0 A} = \frac{q^2}{2\varepsilon_0 A}\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right)\right]$$

$\therefore$ Reduction in energy stored $= U - U'$

$$= \frac{q^2 t\,(1 - 1/\varepsilon_r)}{2\varepsilon_0 A}$$

Case (*ii*). When the potential remains the same.

e.g., when the battery is kept connected to the plates.

(a) Without the slab, energy stored

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2 = \frac{\varepsilon_0 A V^2}{2d}$$

(b) On introducing a dielectric slab of thickness d,

$$U' = \frac{1}{2}C'V^2 = \frac{1}{2}\frac{\varepsilon_r \varepsilon_0 A}{d}V^2 = \frac{\varepsilon_r \varepsilon_0 A V^2}{2d}$$

Thus the energy will *increase* by a factor ε_r.

When a slab of thickness t, $(t < d)$, is introduced, energy stored

$$= U' = \frac{1}{2}C'V^2 = \frac{\varepsilon_0 A V^2}{2[d - t\{1 - (1/\varepsilon_r)\}]}$$

In this case, $U' > U$.

4.11 LOSS OF ENERGY ON SHARING OF CHARGES BETWEEN TWO CAPACITORS

Consider two capacitors of capacitances C_1 and C_2 charged to potentials V_1 and V_2. When they are joined by a wire, they attain a common potential V. (Fig. 4.12)

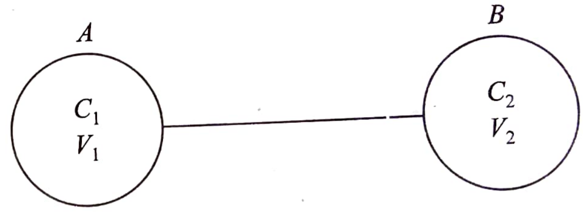

FIG. 4.12

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Total energy of the two capacitors before contact

$$U_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \qquad ...(1)$$

Total energy of the two capacitors after contact

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2$$

$$= \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2} \qquad ...(2)$$

Loss of energy due to contact,

$$U_1 - U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$= \frac{1}{2(C_1 + C_2)}[(C_1 + C_2)(C_1V_1^2 + C_2V_2^2) - (C_1V_1 + C_2V_2)^2]$$

$$= \frac{1}{2(C_1 + C_2)}\left[C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2\right]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}[V_1^2 + V_2^2 - 2V_1V_2]$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2.$$

Since $(V_1 - V_2)^2$ is always positive, U_2 must be less than U_1. Hence there is a *loss* of energy on sharing the charges. The loss of energy appears partly as heat in the connecting wire and partly as light and sound if sparking occurs.

4.12 FORCE OF ATTRACTION BETWEEN PLATES OF A CHARGED PARALLEL PLATE CAPACITOR

Let q be the charge on each of the plates, A be the area and d be the distance between them (Fig. 4.13). Let σ be the surface charge density and E the intensity of electric field between the plates. $E = \sigma/\varepsilon_0$.

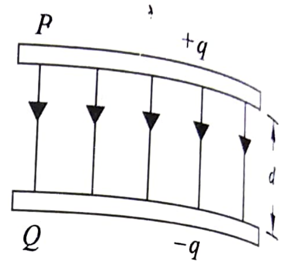

FIG. 4.13

Case (*i*). When the charge on the plates is constant

Force of attraction per unit area between the two plates is equal to the outward electrical force per unit area on the surface of plate P. It is given by

$$p = \frac{\sigma^2}{2\varepsilon_0} = \frac{(q/A)^2}{2\varepsilon_0} = \frac{q^2}{2\varepsilon_0 A^2} = \frac{1}{2}\varepsilon_0 E^2 \qquad \left(\because E = \frac{\sigma}{\varepsilon_0}\right)$$

$$\left.\begin{array}{l}\text{Force of attraction}\\ \text{between the plates}\end{array}\right\} = F = Ap = \frac{q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 E^2 A$$

Case (*ii*). When the P.D. between the plates remains constant

In this case, a battery of EMF V volts is connected across the plates of a capacitor. Now, $E = V/d$.

$$F = \frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2 A$$

Thus we can find F if we know σ or E or V.

Example. *Calculate the force between the plates of a parallel plate capacitor, when the area of the plate is 300 cm² each, the separation is 0.5 cm and they are charged to P.D. 1000 volts.*

Solution. $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}\ m^{-2}$; $A = 300 \times 10^{-4}\ m^2$, $d = 0.5 \times 10^{-2}$ m, $V = 1000$ volts.

$$F = \frac{\varepsilon_0 V^2 A}{2d^2} = \frac{(8.85\times10^{-12})\times(1000)^2\times(300\times10^{-4})}{2\times(0.5\times10^{-2})^2} = 5.3\times10^{-3}\ N$$

4.13 TYPES OF CAPACITORS

(*a*) **Guard Ring Capacitor.** In a parallel plate capacitor, the electric field between the plates is not uniform near the edges. This is called the "*edge effect*" or "*fringing*". The expression $C = \varepsilon_0 A/d$ is only approximate. This is avoided by using a guard ring. The circular insulated plate P is surrounded by a circular coplanar ring G. The inner diameter of G is slightly larger than the diameter of P (Fig. 4.14). The air gap between P and G is very small. The diameter of the plate Q is equal to the outer diameter of G. The field between P and Q is uniform throughout the common area between them. The irregularity in the field occurs at the outer edge of the guard ring. The effective area of the plate = A' = Area of the plate P + $\frac{1}{2}$ Area of the circular air gap between P and G. $C = \varepsilon_0 A'/d$. This is used as an absolute standard of capacitance.

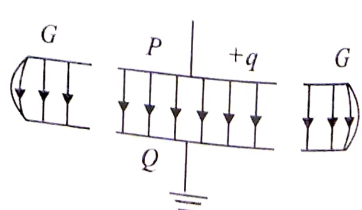

FIG. 4.14

(*b*) **Mica Capacitors.** A schematic diagram of a multiplate capacitor is shown in Fig. 4.15. It consists of a number of parallel plate capacitors in parallel with the alternate metallic foils fixed to one end each. Mica is used as the dielectric. The capacitance of such a system is $C = n\varepsilon_r\varepsilon_0 A/d$ where n is the number of capacitors grouped in parallel, A is the surface area of the plate and d is the thickness of each mica sheet.

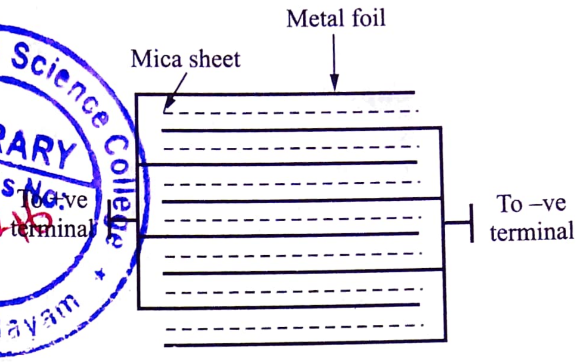

FIG. 4.15

- A fixed capacitor of this type is commonly used in radio sets.
- Fig. 4.16 shows paper capacitor. It is of the parallel plate type. It is made using strips of aluminium foil with waxed paper as dielectric. Two large rectangular metal foils are interleaved between thin sheets of waxed paper. The end *L* of the sandwich is rolled towards *M* to form a cylinder. Each metal foil is connected to a terminal wire. The capacitor is dipped in wax to preserve the insulation.

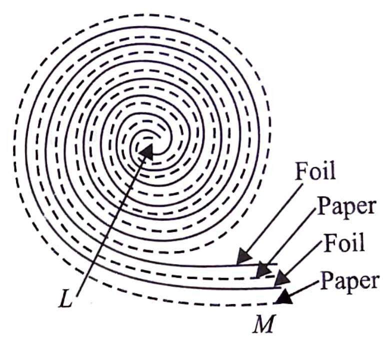

FIG. 4.16

- When capacitors have to be used in high frequency circuits subject to wide variations of temperature, *ceramic capacitors* are preferred. In these, a ceramic material (e.g., hydrous silicate of magnesia or talc) is used as the dielectric. A silver coating is made on the two opposite faces of a thin disc of such a material. The coatings serve as the plates of the capacitor. Leads are attached to each side of the disc. The arrangement is encapsulated in a moisture proof coating. These capacitors are used as bypass capacitors and coupling capacitors in electronic circuits.

(*c*) **Electrolytic Capacitor.** It consists of two aluminium electrodes *A* and *C* dipped in a solution of ammonium borate (Fig. 4.17). On passing a direct current, a very thin film of aluminium oxide is formed on the anode. This film is an insulator. The arrangement can now be used as a capacitor with the anode as one plate, the *solution* as the other plate, and the aluminium oxide film as dielectric. Since the dielectric layer is very thin, the capacitance of this arrangement is very large. This capacitor must be placed only in a D.C. circuit. It cannot be used in an A.C. circuit.

(*d*) **Variable Air capacitor.** It consists of two sets of metal plates, one fixed and the other movable (Fig. 4.18). The fixed set is semi-circular in shape. The movable set is like a cam and rotated with knobs. All the fixed plates are connected to one terminal. All the movable plates are connected to another terminal. Air is the dielectric. By rotating the knobs, the area of overlap between the two

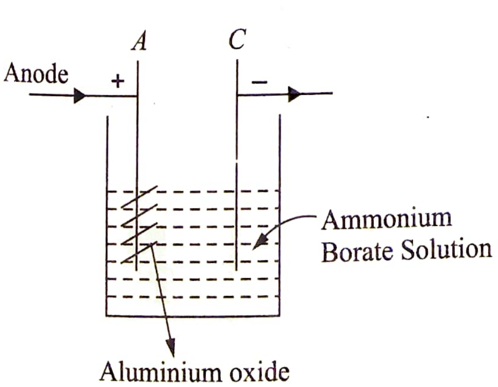

FIG. 4.17

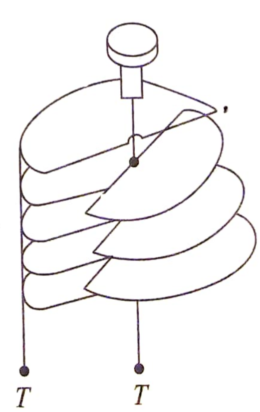

FIG. 4.18

sets of plates is changed. Thus the capacitance of the capacitor changes. These capacitors are widely used in the tuning circuits of radio receivers.

Uses of Capacitors.

(*i*) They are used in the ignition system of automobile engines for eliminating sparking.

(*ii*) They are used in radio circuits for tuning, to reduce voltage fluctuations in power supplies and to increase the efficiency of alternating current power transmission.

(*iii*) They are used to generate and detect electromagnetic oscillations of high frequency.

(*iv*) They serve as useful devices for storing electric energy.

(ii) ELECTROMETERS

4.14 KELVIN'S THE ATTRACTED DISC OR ABSOLUTE ELECTROMETER

Principle. The instrument is based on the force of attraction between the plates of a charged parallel plate capacitor.

Construction. It consists of a guard ring, parallel plate air capacitor. P and Q are two circular metal plates parallel to each other (Fig. 4.19). A guard ring G surrounds the plate P. The plate P serves as the attracted disc. S is a spring attached to P. P can be moved up or down by using the micrometer screw N. P and G are joined by a metal wire and so they are at the same potential. Q can be raised or lowered by means of a micrometer screw M. The distance by which Q is raised or lowered is measured on the scale R.

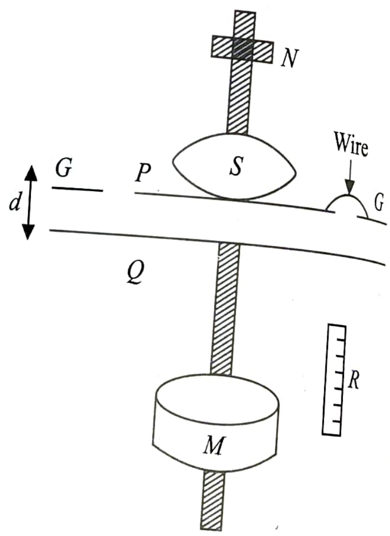

FIG. 4.19

Theory. Let the plates P and Q be connected to potentials V_P and V_q respectively. Let d be the distance between P and Q. Then, the electric field E between the plates P and Q is

$$E = (V_p - V_q)/d \quad ...(1)$$

Therefore, the plate P will experience a force of attraction,

$$F = \frac{\varepsilon_0 E^2}{2} A = \frac{\varepsilon_0 A}{2} \frac{(V_p - V_q)^2}{d^2} \quad ...(2)$$

where A is the "effective area" of the plate P.

$$\therefore \quad V_p - V_q = d\sqrt{\frac{2F}{\varepsilon_0 A}} \quad ...(3)$$

Measurement of Potential Difference between Two given Points

(*i*) The plates P and Q are connected to the earth so that the p.d. between the plates is zero. A small mass m is placed on the plate P. Then P is depressed below the plane of G. The plate P is brought back to the same level as G by adjusting the screw N.

(*ii*) Then the mass is removed. The plate P goes above the level of G.

(*iii*) The plate P is connected to a potential V. Q is connected to one of the potential points, say V_1. The position of Q is adjusted with the help of the screw M, so that P comes in level with the guard ring. Then, force of attraction between P and $Q = mg$.

The reading R_1 of the micrometer is noted. Let d_1 be the distance between P and Q. Then

$$V - V_1 = d_1\sqrt{\frac{2mg}{\varepsilon_0 A}} \quad ...(i)$$

(iv) Finally, the plate Q is connected to the second point whose potential is V_2. The position of Q is again adjusted so that P comes in level with the guard ring. Let the reading on the micrometer be R_2. Let the distance between the plates be d_2. Then

$$V - V_2 = d_2\sqrt{\frac{2mg}{\varepsilon_0 A}} \qquad ...(ii)$$

(i)–(ii) gives,

$$V_2 - V_1 = (d_1 - d_2)\sqrt{\frac{2mg}{\varepsilon_0 A}}$$

$$\therefore \quad V_2 - V_1 = (R_1 - R_2)\sqrt{\frac{2mg}{\varepsilon_0 A}} \qquad (\because d_1 - d_2 = R_1 - R_2)$$

Thus knowing *mg*, *A* and the change in micrometer screw reading, the p.d. between the two points is calculated.

P.D. is obtained in terms of *absolute quantities* like force, length and area. Hence it is called an *absolute electrometer*. It is comparatively less sensitive than the other forms of electrometers.

Determination of Relative Permittivity of a Material (in the form of a parallel slab).

A potential V_p is applied to P and a potential V_q to Q. The distance of Q from P is adjusted so that P is in a level with the guard ring. Let d be the distance between P and Q. Then the capacitance of the capacitor is

$$C = \varepsilon_0 A/d \qquad ...(i)$$

The given slab of thickness t and of the same area as the plates, is placed on the plate Q. The effective air distance between the plates decreases by an amount $t\left(1 - \frac{1}{\varepsilon_r}\right)$. The force on the plate P increases and it moves down. The plate Q is moved down until the plate P goes back to its original position. Let x be the distance by which the plate Q is moved.

Now,

$$C = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right) + x\right]} \qquad ...(ii)$$

The capacitance of the capacitor in the two parts (with and without the slab) are equal. Therefore,

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right) + x\right]} \qquad \text{or} \quad x = t - \frac{t}{\varepsilon_r}$$

$$\therefore \quad \varepsilon_r = \frac{t}{t - x} \qquad ...(iii)$$

The thickness of the slab t is measured by bringing the plates in contact with and without the slab in between these two. Thus ε_r is calculated.

4.15 THE QUADRANT ELECTROMETER

Construction. It consists of four similar hollow metallic quadrants *AA* and *BB* supported separately on amber insulating stands (Fig. 4.20). The opposite pairs of quadrants *AA* and *BB* are connected together by fine copper wires. A paddle-shaped aluminium needle *N* with two wings *CC* is suspended symmetrically between the four quadrants by means of a torsion fibre made of phosphor bronze. The deflection of the needle is measured with the help of a mirror M using the lamp and scale arrangement. The whole arrangement is enclosed in an earthed brass case provided with glass windows to make observations. The base of the instrument is provided with levelling screws.

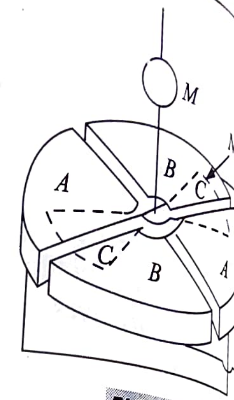

FIG. 4.20

Working. The needle N is usually kept at a constant high potential V_n by connecting it to the positive pole of a H.T. battery whose negative is earthed. When the two pairs of quadrants, AA and BB, are charged to the same potential, the needle rests symmetrically between them. If they are given different potentials, say V_a and V_b such that $V_a > V_b$, then the needle deflects from A quadrants to B quadrants. This deflection is opposed by the torsion of the suspension. At equilibrium, the torsional couple is equal and opposite to the deflecting couple. The deflection θ is proportional to the potential difference $(V_a - V_b)$.

Theory. Let V_n be the constant high potential applied to the needle (Fig. 4.21). Let the pair of quadrants AA and BB be connected to points at potentials V_a and V_b respectively. Let $V_a > V_b$. The needle will get deflected through an angle θ from the pair AA (at higher potential) to the pair BB (at lower potential).

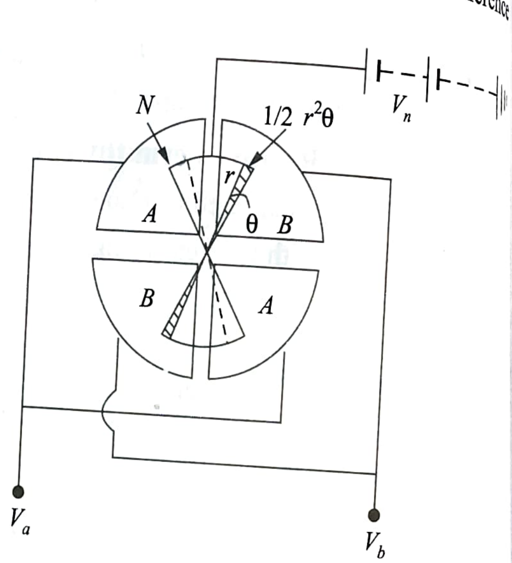

FIG. 4.21

The quadrants AA and the needle N form a parallel-plate capacitor at a potential difference $(V_n - V_a)$. The quadrants BB and the needle form another capacitor at a potential difference $(V_n - V_b)$.

Let r be the radius of the needle. With the deflection of the needle through an angle θ, a surface area $\frac{1}{2}r^2\theta$ of the needle is transferred from the quadrants AA to BB. Since the needle has two arms and two faces, the total area transferred from AA to BB is given by

$$A = 4 \times \frac{1}{2} r^2\theta = 2r^2\theta$$

The shift in area causes an increase in the capacitance of the $B - N$ capacitor given by

$$\delta C = \frac{\varepsilon_0 A}{d} = \frac{2r^2\theta\varepsilon_0}{d}$$

where d is the distance between the surface of the needle and the top or bottom of the quadrants.

The capacitance of the $A - N$ capacitor decreases by the same amount.

Therefore, the increase in the energy of the $B - N$ capacitor

$$= \frac{1}{2} \times \delta C \times (\text{P.D.})^2$$

$$= \frac{1}{2} \times \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_b)^2 = \frac{r^2\theta\varepsilon_0}{d}(V_n - V_b)^2.$$

Similarly, the decrease in the energy of the $A - N$ capacitor $= \frac{r^2\theta\varepsilon_0}{d}(V_n - V_a)^2.$

$\therefore$ net increase in the energy of the system $= \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]$

In addition to this net gain in electrical potential energy, work has also to be done in twisting the suspension fibre by an angle θ. It is given by $\frac{1}{2}c\theta^2$ where c is the restoring couple per unit twist of the suspension fibre.

$$\therefore \quad \text{total energy gain} = \frac{1}{2}c\theta^2 + \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]. \qquad ...(i)$$

As the capacitance of $B - N$ capacitor increases by δC, it draws a charge $\delta C \times (V_n - V_b)$ from the source.

$\therefore$ Energy drawn from the source at a constant p.d. $(V_n - V_b)$ is

$$= \text{charge} \times \text{potential difference}$$

$$= \{\delta C \times (V_n - V_b)\} \times (V_n - V_b)$$

$$= \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_b)^2.$$

Similarly, the $A - N$ capacitor, whose capacitance decreases by δC, restores to the source an amount of energy

$$= \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_a)^2.$$

$\therefore$ net energy drawn from the source

$$= \frac{2r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]. \qquad ...(ii)$$

This must be equal to the total energy gained by the electrometer. Hence equating (i) and (ii), we get

$$\frac{1}{2}c\theta^2 + \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2] = \frac{2r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]$$

or

$$\frac{1}{2}c\theta^2 = \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]$$

$$= \frac{2r^2\theta\varepsilon_0}{d}(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right]$$

or

$$\theta = \frac{4r^2\varepsilon_0}{cd}(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right]$$

$$\therefore \quad \theta = k(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right], \qquad ...(iii)$$

where $k = \frac{4r^2\varepsilon_0}{cd}$

There are two ways of using the electrometer.

(*a*) **Heterostatic use.** The needle is charged to a very high potential which is very large as compared with V_a or V_b. Thus $(V_a + V_b)/2$ is negligible as compared to V_n.

Eq. (iii) reduces to $\theta = kV_n(V_a - V_b)$

i.e., $\theta \propto (V_a - V_b)$.

Hence, the deflection of the needle is directly proportional to the difference of potential between the quadrants AA and BB. This method is used for the determination of small potential difference only.

(*b*) **Idiostatic use.** The needle voltage V_n is made equal to V_a by connecting the needle to the *AA* quadrants. Smaller potential is given to *BB* pair.

Eq. (*iii*) reduces to $\theta = \frac{k}{2}(V_a - V_b)^2$

So both steady and alternating potential differences can be measured since $(V_a - V_b)^2$ is always positive.

Measurement of ionisation current. The quadrant electrometer used heterostatically is the most suitable instrument to measure ionisation currents which are of the order of 10^{-12} *A*.

Experimental Arrangement. The arrangement consists of an earthed metal chamber *C* containing some gas and two metal plates *P* and *Q* insulated from the chamber (Fig. 4.22). The plate *P* is connected to the positive terminal of a high-tension battery whose negative terminal is earthed. The plate *Q* is connected to one pair of quadrants *AA* of the electrometer. The other pair *BB* is earthed. The needle of the electrometer is charged to a high positive potential.

Working. When the gas in the chamber is ionised, negative ions move toward the plate *P*, the positive ions move toward *Q* and thus an ionisation current is set up. Let *q* be the charge on the plate *Q* at any instant *t*. Then the ionisation current

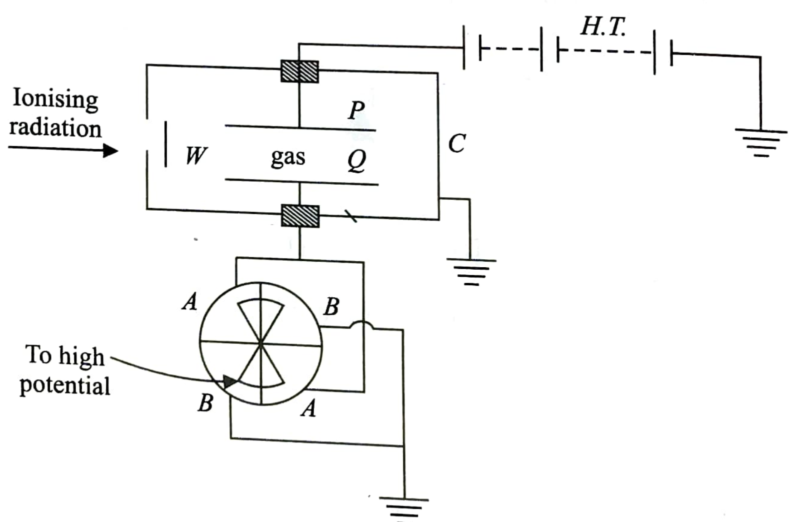

FIG. 4.22

$$i = dq/dt. \quad \text{...(1)}$$

Let *C* be the capacitance of the electrometer together with the plate *Q* connected to it and *V* the potential at the instant *t*. Then $q = CV$ and Eq. (1) reduces to

$$i = \frac{d}{dt}(CV) = C\frac{dV}{dt}.$$

If θ is the deflection of the needle at any instant,

$$\theta = kV, \text{ where } k \text{ is a constant}$$

or

$$V = \frac{\theta}{k}.$$

$\therefore$

$$i = C\frac{dV}{dt} = \frac{C}{k}\left(\frac{d\theta}{dt}\right). \quad \text{...(2)}$$

$d\theta/dt$ is found from the slope of the graph between θ and *t*. The capacitance *C* of the electrometer is found from a separate experiment. The constant *k* is also previously determined by calibrating the electrometer by applying known potential differences across the quadrants, and plotting a graph between θ and V. The slope of this graph gives k. Hence i can be found out.

4.16 DIELECTRIC CONSTANT OF A SOLID-HOPKINSON'S NULL METHOD

Experimental Arrangement. This method is applicable to solids available in the form of a slab. Connections are made as in Fig. 4.23. The needle of the quadrant electrometer is given a constant potential V using a battery. C_1 is a variable capacitor. C_2 is a guard ring capacitor in which the lower earthed plate is movable. The distance through which the lower plate of C_2 is moved can be measured using a micrometer screw M. The mid-point of the battery B_1 is earthed so that its end points are kept at potential differences $+V$ and $-V$ with respect to the earth. Using this arrangement the capacitors C_1 and C_2 can be charged to equal and opposite potentials by pressing the key K. On releasing the key, the capacitors charge the quadrant electrometer. If their capacitances are also made equal by suitably varying C_1, their charges will become equal and the electrometer will show zero deflection. So the method is called a null method.

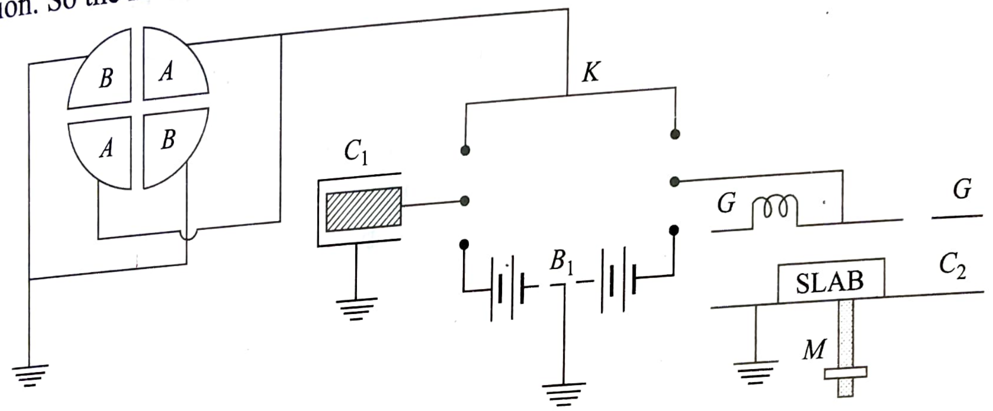

FIG. 4.23

Working. The capacitance of the variable capacitor C_1 is adjusted till on pressing and releasing the key, there is no deflection in the quadrant electrometer. Now the position of the lower plate of C_2 is read on the micrometer scale.

The dielectric slab of thickness t and dielectric constant (relative permittivity) ε_r is introduced between the plates of C_2. As a result the capacitance of C_2 increases. The lower plate of C_2 is moved till the electrometer shows null deflection. Now the capacitance of C_2 is equal to that of C_1. The distance x through which the lower plate of C_2 has been moved is measured using the micrometer screw. Let A be the effective area of guard-ring capacitor plates and d the distance between them.

Calculation. Capacitance of C_2 without the dielectric $= \dfrac{\varepsilon_0 A}{d}$

Capacitance of C_2 with the dielectric $= \dfrac{\varepsilon_0 A}{\left(d + x - t + \dfrac{t}{\varepsilon_r}\right)}$

But the capacitance in the two cases is the same.

Hence,
$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{\left(d + x - t + \dfrac{t}{\varepsilon_r}\right)}$$

i.e.,
$$d = d + x - t + \frac{t}{\varepsilon_r}$$