17UMAE01

OPERATIONS RESEARCH

B.Sc MATHEMATICS

V - SEMESTER

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Unit - II

Introduction - Balanced and unbalanced T.P, Feasible solution - Basic feasible solution - optimum solution - degeneracy in a T.P - Mathematical formulation - North west corner rule - vogell's approximation method (unit Penalty method) Method of Matrix Minima (least cost method) - problems - algorithm of optimality test (modi-Method) - problems. Introduction - Definition of Assignment problem, balanced and unbalanced assignment problems. Introduction - Definition of problem - Mathematical formulation and solution of an assignment problem (Hungarion method) - degeneracy in an assignment problem - problems.

Unit - II

Transportation problem

Aim:

The aim of the transportation problem is to find the minimum cost for the given problem.

Mathematical form of T.P:

Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \, c_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i \, , \; i = 1, 2, \ldots, m.$

$\sum_{i=1}^{m} x_{ij} = b_j \, , \; j = 1, 2, \ldots, n.$

and all $x_{ij} \geq 0$ for all i and j.

Balanced T.P:

A T.P is said to be balanced if the sum of supply is equal to sum of demand.

unbalanced T.P:

A T.P is said to be unbalanced if the sum of supply is not equal to sum of demand.

Methods for finding basic feasible solution in T.P:

- North west corner method (NWC)
- Least cost method (LCM) (or) Matrix Minima method.
- Vogel's Approximation method (VAM)

1. Solve the T.P using (i) (NWC) ii). (LCM) (iii) VAM

	A	B	C	D	Supply
E	11	13	17	14	250
F	16	18	14	10	300
G	21	24	13	10	400
Demand	200	225	275	250	

Sol:

Sum of supply = 950

sum of demand = 950

sum of supply = sum of demand

950 = 950

The given T.P is balanced.

Number of Basic cells

$= m+n-1$

$= 3+4-1$

$= 7-1$

$= 6$

i). NWC

(250-200 = 50)

[200] ~~11~~	13	17	14	250
~~16~~	18	14	10	300
~~21~~	24	13	10	400
200	225	275	250	

[50] ~~13~~	17	14	50
~~18~~	14	10	300
~~24~~	13	10	400
(225-50= 175) 225	275	250	

(300-175= 125)

[175] ~~18~~	14	10	300
~~24~~	13	10	400
175	275	250	

[125] ~~14~~	~~10~~	125
13	10	400
(275-125= 150) 275	250	

[150] ~~13~~	10	400 (400-150= 250)
150	250	

[250] 10	250
250	

[200] 11	[50] 13	17	14
16	[175] 18	[125] 14	10
21	24	[150] 13	[250] 10

The minimum transportation cost

$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 14 \times 125 + 13 \times 150 + 10 \times 250$

$=$ RS. 12, 200.

ii). LCM:

11	13	17	~~14~~	250
16	18	14	250 10	300 (300-250=50)
21	24	13	~~10~~	400
200	225	275	250	

200 11	13	17	250 (250-200=50)
16	18	14	50
21	24	13	400
200	225	275	

50 13	17	50
18	14	50
24	13	400
225 (225-50=175)	275	

18	14	50
24	275 13	400 (400-275=125)
175	275	

50 18	50
24	125
175 (175-50=125)	

125 24	125
125	

200 11	50 13	17	14
16	50 18	14	250 10
21	125 24	275 13	10

The minimum transportation cost

$= 11\times 200 + 13\times 50 + 18\times 50 +$
$10\times 250 + 13\times 275 + 24\times 125$

= RS. 12,825.

iii). VAM:

200 11	13	17	14	250 (2) (250-200=50)
16	18	14	10	300 (4)
21	24	13	10	400 (3)
200	225	275	250	
(5)	(5)	(1)	(0)	

50 13	17	14	50	(1)
18	14	10	300	(4)
24	13	10	400	(3)
(225-50=175) 225	275	250		
(5)	(1)	(0)		

175 18	14	10	300 (4) (300-175=125)
24	13	10	400 (3)
175	275	250	
(6)	(1)	(0)	

14	125 10	125 (4)
13	10	400 (3)
275	250 (250-125=125)	
(1)	(0)	

13	125 10	400 (3)
275	125	(400-275=175)

275 13	275
275	

200 11	50 13	17	14
16	175 18	14	125 10
21	24	275 13	125 10

The minimum transportation cost $= 11\times 200 + 13\times 50 + 18\times 175 +$
$10\times 125 + 13\times 275 + 125\times 10$

= RS. 12,075

2. Solve the T.P by using i). NWC ii). LCM iii). VAM.

	D	E	F	Supply
	1	2	6	7
	0	4	2	12
	3	1	5	11
Demand	10	10	10	

Sum of supply = 30

sum of demand = 30

sum of supply = sum of demand

30 = 30

The given T.P is balanced

i). NWC

7 1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

No. of basic cells

$= m+n-1$

$= 3+3-1$

$= 5$

3 0	4	5	12
3	1	5	11
3	10	10	

9 4	2	9
1	2	11
10	10	

1 1	5	11
1	10	

10 5	10
10	

7 1	2	6
3 0	9 4	2
3	1 1	10 5

∴ Minimum T cost

$= 1\times7 + 0\times3 + 4\times9 + 1\times1 + 5\times10$

$=$ RS 94

II). LCM:

1	2	5	7
10 0	4	2	12
3	1	5	11
10	10	10	

2	6	7
4	2	2
10 1	5	11
10	10	

6	7
2 2	2
5	1
10	

6	7
1 5	1
8	

7 6	7
7	

1	2	7 6
10 0	4	2 2
3	10 1	1 5

Minimum T. Cost

$= 0\times10 + 1\times10 + 6 + 7 + 2\times2 + 5\times7$

$=$ RS. 61

III). VAM

1	2	6	7	(1)
0	4	10 2	12	(2)
3	1	5	11	(3)
10	10	10		
(2)	(1)	(3)		

Order

1	2	7	(1)
2 0	4	2	(4)
3	1	11	(2)
10	10		
(2)	(1)		

1	2	7	(1)
3	10 1	11	(2)
8	10		
(2)	(1)		

7 1	7
3	1
8	
(2)	

1 3	1
1	

7 1	2	6
2 0	4	10 2
1 3	10 1	5

$\therefore$ Minimum T. Cost

$= 1\times7 + 0\times2 + 3\times1 + 1\times10 + 2\times10$

$=$ RS. 40

2 very important

MODI METHOD:

To find the optimum solution:

1. Solve the following T.P and find the optimum soln:

	A	B	C	D	
	11	20	7	8	50
	21	16	20	12	40
	8	12	18	9	70
Demand	30	25	35	40	

Sum of supply $= 160$

Sum of demand $= 130$

Sum of supply ≠ Sum of demand

The total supply is greater than the total demand.

The greater problem is unbalanced, to convert into a balanced, we introduce a dummy destination E with [illegible] unit.

Transportation cost and having a demand 160 - 130 = 30 unit.

Inference the given problem becomes

A	B	C	D	E	
11	20	7	8	0	50
21	16	20	12	0	40
8	12	18	9	0	70
30	25	35	40	30	

VAM:

11	20	7	8	0	50	(7)
21	16	20	12	^{10}0	40	(12)
8	12	18	9	0	70	(8)
30	25	35	40	30		
(3)	(4)	(8)	(1)	(0)		

11	20	^{15}7	8	50	(1)
21	16	20	12	10	(4)
8	12	18	9	70	(1)
30	25	35	40		
(3)	(4)	(11)	(1)		

11	20	8	15	(3)
21	16	12	10	(4)
8	^{25}12	9	70	(1)
30	25	40		
(3)	(4)	(1)		

11	8	15	(3)
21	^{10}12	10	(4)
8	9	45	(1)
30	40		
(1)	(1)		

11	8	15	(1)
^{10}8	9	45	(1)
30	30		
(3)	(1)		

^{15}8	15
9	15
30	
(1)	

^{15}9	15
15	

Minimum T cost = 7x35+ 8x15x18x10+0x30+8x30+ 18x25+9x15 = RS.1.160

11	20	^{16}7	^{15}8	0
21	16	20	^{10}12	^{10}0
^{10}8	^{25}12	18	^{15}9	0

11	20	^{16}7	^{15}8	0	U_1
21	16	20	^{10}12	^{10}0	U_2
^{10}8	^{25}12	18	^{15}9	0	U_3
V_1	V_2	V_3	V_4	V_5	

To find occupied all:

$$u_i + v_j = c_{ij}$$

$u_1 + v_3 = 7$ Let $u_3 = 0$ $u_2 + v_4 = 12$

$u_1 + v_4 = 8$ $u_3 + v_1 = 8$ $u_2 = 12 - 9$

$u_2 + v_4 = 12$ $v_1 = 8$ $u_2 = 3$

$u_2 + v_5 = 0$ $u_3 + v_2 = 12$ $u_2 + v_5 = 0$

$u_3 + v_1 = 8$ $v_2 = 12$ $v_5 = -3$

$u_2 + v_2 = 12$ $u_3 + v_4 = 9$ $u_1 + v_3 = 7$

$u_3 + v_4 = 9$ $v_4 = 9$ $-1 + v_3 = 7$

$u_1 + v_4 = 8$ $v_3 = 8$

$u_1 = 8 - 9$

$u_1 = -1$

11	20	15 7	15 8	0	u_1 ($u_1 = -1$)
21	16	20	10 12	10 0	u_2 ($u_2 = 3$)
10 8	25 12	18	18 9	0	u_3 ($u_3 = 0$)
v_1 ($v_1 = 8$)	v_2 ($v_2 = 12$)	v_3 ($v_3 = 8$)	v_4 ($v_4 = 9$)	v_5 ($v_5 = 3$)	

To find unoccupied all:

$$z_{ij} - c_{ij} = u_i + v_j - c_{ij}$$

$z_{11} - c_{11} = u_1 + v_1 - c_{11} = -1 + 8 - 11 = -4$

$z_{12} - c_{12} = u_1 + v_2 - c_{12} = -1 + 12 - 20 = -1$

$z_{15} - c_{13} = u_1 + v_2 - c_{13} = -1 - 3 - 0 = -4$

$z_{21} - c_{21} = u_2 + v_1 - c_{21} = 3 + 8 - 21 = -10$

$z_{22} - c_{22} = u_2 + v_2 - c_{22} = 3 + 12 - 16 = -1$

$z_{23} - c_{23} = u_2 + v_3 - c_{23} = 3 + 8 - 20 = -9$

$z_{33} - c_{33} = u_3 + v_3 - c_{33} = 0 + 8 - 18 = -10$

$z_{35} - c_{35} = u_3 + v_5 - c_{35} = 0 + (-3) - 0 = -3$

Since all $z_{ij} - c_{ij} \leq 0$

The current basic feasible soln is optimum

$\therefore$ The optimum soln is

$x_{13} = 7$, $x_{14} = 8$, $x_{24} = 12$, $x_{23} = 0$,

$x_{31} = 8$, $x_{32} = 12$, $x_{34} = 9$.

The initial transportation cost = RS. 1,160.

2. Solve the following T.P

A	B	C	Supply
50	30	220	1
90	45	170	3
250	200	50	4
Demand 4	2	2	

Sol:

Sum of supply = 8
Sum of demand = 8
∴ Sum of supply = sum of demand
8 = 8

The given T.P is balanced

50	30	220	1 (20)
90	45	170	3 (45)
250	200	50	4 (150)
4 (40)	2 (15)	2 (120)	

$=2(V)$

50	30	1 (20)
^{3}90	45	3 (45)
250	^{2}200	2 (50)
4 (40)	2 (15)	

^{1}50	1
90	3
4 (40)	

^{3}90	3
3	

^{1}50	30	220
^{3}90	45	170
^{b}250	^{2}200	^{2}50

Minimum T. cost = 50×1 + 90×3 + 200×2 + 50×2 + 250×b

= RS. 820

H.W

3. Solve the following T.P by using i). NWC ii). LCM iii) VAM

	A	B	C	Supply
	2	7	4	5
	3	3	1	8
	5	4	7	7
	1	6	2	14
Demand	7	9	18	

Sum of supply = 34
Sum of demand = 34
Sum of supply = sum of demand
34 = 34
The given T.P is balanced

ii). NWC

$^{5}2$ 7 4 5
3 3 1 8
5 4 7 7
1 6 2 14
7 9 18

$^{2}3$ 3 1 8
5 4 7 7
1 6 2 14
2 9 18

$^{6}3$ 1 6
4 7 7
6 2 14
9 18

$^{3}4$ 7 7
6 2 14
3 18

$^{4}7$ 4
2 14
18

$^{14}2$ 14
14

$^{5}2$ 7 4 5
$^{2}3$ $^{6}3$ 1 8
5 $^{3}4$ $^{4}7$ 7
1 6 $^{14}2$ 14
7 9 18

Minimum T. cost
= 2×5 + 3×2 + 3×6 + 4×3 + 7×4 + 2×14
= RS.102

iii). LCM

2 7 4 5
3 3 $^{8}1$ 8
5 4 7 7
1 6 2 14
7 9 18

2 7 4 5
5 4 7 7
$^{7}1$ 6 2 14
7 9 10

7 4 5
4 7 7
5 $^{7}2$ 7
9 10

7 $^{3}4$ 5
4 7 7
9 3

7 2
$^{7}4$ 7
9

$^{2}7$ 2
2

2 7 $^{3}4$
3 3 $^{8}1$
5 $^{7}4$ $^{2}7$
$^{7}1$ 6 $^{7}2$

Minimum T. cost
= 4×3 + 1×8 + 4×7 + 7×2 + 1×7 + 2×7
= RS. ~~79~~ 83

Definitions:

Feasible Solution:

A set of non-negative values x_{ij}, $i = 1, 2, \ldots m$; $j = 1, 2, \ldots n$. That satisfies the constrains is called a feasible salution.

Basic feasible Solution:

A feasible solution to a $(m \times n)$ transportation problem that contains no more than $m+n-1$ non-negative is called Basic feasible solution.

Definition:

A basic feasible salution to a $(m \times n)$ transportation problem is said to be a non-degenerate basic feasible salution if it contains exactly $m+n-1$ non-negative allocation is independent position.

Definition : Degenerate Basic feasible solution

A Basic feasible solution that contains less than $m+n-1$ non-negative allocation is said to be a degenerate basic feasible solution.

Definition : optimum solution :-

A feasible solution is said to be optimal solution, if it minimise the total transportation cost.

METHOD : 1

North West corner Rule

Step : 1

The first assignment is made in the cell occuping the upper left - hand (North west corner of the transportation table. The maximum possible amount is allocated there

That PS $x_{11} = \min \{a, b\}$

case (i)

If $\min \{a_1, b_1\} = a_1$, then put $x_{11} = a_1$, decrease b_1 by a_1 and moves vertically to the 2nd row. (i.e) to the cell (2,1) cross out the first row.

case (ii).

If $\min \{a_1, b_1\} = b_1$, then put $x_{11} = b_1$, and decrease a_1 by b_1 and move horizontally right (i.e) to the cell (1,2) cross out the first colum.

case (iii).

If $\min \{a_1, b_1\} = a_1 = b_1$, then put $x_{11} = a_1 = b_1$ and move diagonally to the cell (2,2) cross out the first row and first column.

Step : 2

Repeat the procedure untill all the aim requirements are satisfied.

METHOD : 2

Least cost Method (or) Matrix minima method (or) lowest cost entry method.

Step : 1

Identify the cell with smallest cost and allocate

$$x_{ij} = \min\{a_i, b_j\}$$

case (i):

If $\min\{a_i, b_j\} = a_i$, then put $x_{ij} = a_i$, cross out the i^{th} row and decrease b_j by a_i, go to step (2).

case (ii):

If $\min\{a_i, b_j\} = b_j$, then put $x_{ij} = b_j$, cross out the j^{th} column and decrease a_i by b_j, cross out either i^{th} row or j^{th} column but not both. go to step (2).

case (iii):

If $\min\{a_i, b_j\} = a_i = b_j$. then put $x_{ij} = a_i = b_j$, If $\min\{a_i, b_j\} = a_i = b_j$ cross out either i^{th} row or j^{th} column but not both & go to step (2).

Step : 2

Repeat step (1) for the resulting. reduced transportation table untill all the aim requirements are statisfied.

METHOD: 3

Vogel's approximation method (or) unit cost penalty Method.

Step : 1

Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step : 2

Identify the row or column with largest penalty. If a tie occurs, break the tie arbitarily choose the cell with smallest cost in that selected row (or) column and allocate as much as possible to this cell and cross and the satisfied row or column and go to step (3).

Step:3

Again compute the column and row penalities for the reduced transportation table and then go to step (2).

Repeat the procedure untill all the aim requirements are statisfied.

The Assignment Algorithm:

Step 1:

Subtract the minimum cost of each row of the cost matrix from all the elements of the respective row. Then, modify the resulting matrix by subtracting the minimum cost of each column from all the elements of the respective columns, obtaining the starting matrix.

Step : 2

Draw the least possible number of horizontal and vertical lines to cover all the zeroes of the starting table. Let the number of these lines be N. There now arise two cases.

i). N = n, the order of the cost matrix. In this case an optimum assignment has been attained.

ii). N < n in this case go to next step.

Step : 3

Determine the smallest cost in the starting table, not covered by the N lines subtract this cost from all the surviving (un-covered) elements of the starting matrix and add the same to all these elements of the starting matrix which are lying at the intersection of horizontal and vertical lines, thus obtaining the second modified cost matrix.

Step : 4

Repeat steps 1, 2 and 3 until we get N=n.

Step : 5

Examine the rows successively untill a row with exactly one unmarked zero is found. Enclose this zero inside a circle (a) and an assignment will be made Here make a cross (x) in the cells of all other zeroes lying in the column of the incircled zero to show that they cannot be considered for fucture assignment continue in the manner until all the rows have been taken care of.

Step : 6

Examine the columns successively, untill a column with exactly one unmarked zero is found encircle that zero as an assignment will be made there mark a cross (x) in the cells of all other zero lying in the row of encircled zero continue in this way untill all the columns have been taken care of.

Step : 7

Repeat steps 5 and 6 successively untill one of the following arises. (i). no unmarked zero is left and (ii). There lie more than one unmarked zeroes in one column or row. In case (i) algorithm steps In case (ii). encircle one of the unmarked zeroes arbitrarily and mark a cross in the cells of remaining zeroes in its row and column. Repeat the process until no unmarked zero is left in the cost matrix.

Step : 8

We now have exactly one encircled zero in each row and each column of the cost matrix the assignment schedule corresponding to these zeroes is the optimum (maximal) assignment.

Note:

The above iterative method of determining an assignment schedule is known as Hungarian assignment method.

Assignment Problem: (Hongarian Method) 2m

Assignment problem is used to find the minimum time required to complete the project with the assignment schedule.

1. Solve the assignment problem:

$$\begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 10 & 25 & 15 & 20 \\ 15 & 30 & 5 & 15 \\ 35 & 20 & 12 & 24 \\ 17 & 25 & 24 & 20 \end{pmatrix} \end{array}$$

Soln:

Here no. of rows = no. of columns

∴ The given problem is balanced.

Step : 1

Row difference

$$\begin{pmatrix} 0 & 15 & 5 & 10 \\ 10 & 25 & 0 & 10 \\ 23 & 8 & 0 & 12 \\ 0 & 8 & 7 & 3 \end{pmatrix}$$

Step : 2

column difference

$$\begin{pmatrix} \boxed{0} & 7 & 5 & 7 \\ 10 & 17 & \boxed{0} & 7 \\ 23 & \boxed{0} & \cancel{0} & 9 \\ \cancel{0} & \cancel{0} & 7 & \boxed{0} \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is

$\mathrm{I} \to A$, $\mathrm{II} \to C$, $\mathrm{III} \to B$, $\mathrm{IV} \to D$

Min. time $= 10 + 5 + 20 + 20 = 55$ days.

2. Solve the assignment pbm:

$$\begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 18 & 26 & 17 & 11 \\ 13 & 28 & 14 & 26 \\ 38 & 19 & 18 & 15 \\ 19 & 26 & 24 & 10 \end{pmatrix} \end{array}$$

Soln:

Here no. of rows = no. of column

∴ The given problem is balanced

Step : 1

Row difference

$$\begin{pmatrix} 7 & 15 & 16 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{pmatrix}$$

✓ Non - assigning row

↓

✓ zero consisting column

↓

assigning row

✓ marked column

unmarked row

Step : 2

column difference

$$\begin{pmatrix} 7 & 11 & 15 & \boxed{0} \\ \cancel{\boxed{0}} & \cancel{11} & \cancel{0} & \cancel{13} \\ \cancel{23} & \cancel{\boxed{0}} & \cancel{2} & \cancel{0} \\ 9 & 12 & 13 & \cancel{0} \end{pmatrix} \begin{matrix} \checkmark \\ \\ \\ \checkmark \end{matrix}$$

Step : 3

$$\begin{pmatrix} 2 & 6 & \boxed{0} & \cancel{0} \\ \boxed{0} & 11 & \cancel{0} & 18 \\ 23 & \boxed{0} & 2 & 5 \\ 4 & 7 & 8 & \boxed{0} \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is

I → C, II → A, III → B, IV → D

Min time = 17 + 13 + 19 + 10 = 59 days.

3. Solve the assignment pbm:

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline I & 11 & 17 & 8 & 16 \\ II & 9 & 7 & 12 & 6 \\ III & 13 & 16 & 15 & 12 \\ IV & 14 & 10 & 12 & 11 \end{array}$$

Soln: Here no. of rows = no. of columns

∴ The given pbm is balanced.

Step : 1

Row Difference

$$\begin{pmatrix} 3 & 9 & 0 & 8 \\ 3 & 1 & 6 & 0 \\ 1 & 4 & 3 & 0 \\ 4 & 0 & 2 & 1 \end{pmatrix}$$

Step : 2 column Difference

$$\begin{pmatrix} 2 & 9 & \boxed{0} & 8 \\ 2 & 1 & 6 & \boxed{0} \\ \boxed{0} & 4 & 3 & \cancel{0} \\ 3 & \boxed{0} & 2 & 1 \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is

$I \to C,\ II \to D,\ III \to A,\ IV \to B$

Min Time $= 8+6+13+10 = 37$ days.

4. Solve the assignment pbm:

$$\begin{array}{c} \\ I \\ II \\ III \\ IV \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 1 & 4 & 6 & 3 \\ 9 & 7 & 10 & 9 \\ 4 & 5 & 11 & 7 \\ 8 & 7 & 8 & 5 \end{pmatrix} \end{array}$$

Here no. of rows = no. of columns

$\therefore$ The given pbm is balanced.

Step : I

Row Difference

$$\begin{pmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 7 & 3 \\ 3 & 2 & 3 & 0 \end{pmatrix}$$

Step : 2

column Difference

$$\begin{pmatrix} \boxed{0} & 3 & 2 & 2 \\ 2 & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & 1 & 4 & 3 \\ 3 & 2 & \boxed{0} & \cancel{0} \end{pmatrix}$$

Step : 3

$$\begin{pmatrix} \boxed{0} & 2 & 1 & 1 \\ 3 & \cancel{0} & \boxed{0} & 2 \\ \boxed{0} & \boxed{0} & 3 & 2 \\ 4 & 2 & \cancel{0} & \boxed{0} \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is

$I \to A,\ II \to C,\ III \to B,\ IV \to D$

Min time $= 1+10+5+5 = 21$ days.

5. Solve the assignment pbm:

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Soln:

Here no. of rows = no. of columns

$\therefore$ The given pbm is balanced.

Step : 1

Row Difference

$$\begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Step : 2

column Difference

$$\begin{pmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix} \begin{matrix} \checkmark \\ \checkmark \\ \checkmark \\ \end{matrix}$$

Step : 3

$$\begin{pmatrix} \boxed{0} & 1 & 5 & 9 \\ \cancel{0} & \boxed{0} & 4 & 6 \\ \cancel{0} & \cancel{0} & 4 & 7 \\ 5 & \cancel{0} & \boxed{0} & \cancel{0} \end{pmatrix} \begin{matrix} \checkmark \\ \checkmark \\ \checkmark \\ \end{matrix}$$

Step : 4

$$\begin{pmatrix} \boxed{0} & 1 & 1 & 5 \\ \cancel{0} & \boxed{0} & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & \boxed{0} & 3 \\ 9 & 4 & \cancel{0} & \boxed{0} \end{pmatrix}$$

Here each row and column having only one assignment

Assignment schedule is

$I \to A$, $II \to B$, $III \to C$, $IV \to D$

Min time $= 18 + 13 + 19 + 0 = 50$ days.

6. Solve the assignment pbm:

	1	2	3	4
A	5	3	2	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	8

Here no. of rows = no. of columns

$\therefore$ The given pbm is balanced.

Step : 1

Row Difference

$$\begin{pmatrix} 3 & 1 & 0 & 6 \\ 5 & 7 & 0 & 2 \\ 2 & 0 & 1 & 3 \\ 0 & 2 & 2 & 3 \end{pmatrix}$$

Step : 2

column Difference

$$\begin{pmatrix} 3 & 1 & \boxed{0} & 3 \\ 5 & 7 & \cancel{0} & 1 \\ 2 & \boxed{0} & 1 & \cancel{0} \\ \boxed{0} & 2 & 2 & \cancel{0} \end{pmatrix} \begin{matrix} \checkmark \\ \checkmark \\ \\ \end{matrix}$$

Step : 3

$$\begin{pmatrix} 2 & \boxed{0} & \cancel{0} & 2 \\ 4 & 6 & \boxed{0} & \cancel{0} \\ 2 & \boxed{0} & 2 & \cancel{0} \\ \boxed{0} & 2 & 3 & \cancel{0} \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is

$A \to 2, \ B \to 3, \ C \to 4, \ D \to 1$

Min time $= 3 + 2 + 7 + 5 = 17$ days.

7. Solve the assignment pbm:

$$\begin{array}{c|ccccc} & V & W & X & Y & Z \\ \hline A & 3 & 5 & 10 & 15 & 8 \\ B & 4 & 7 & 15 & 18 & 8 \\ C & 8 & 12 & 20 & 20 & 12 \\ D & 5 & 5 & 8 & 10 & 6 \\ E & 10 & 10 & 15 & 25 & 10 \end{array}$$

soln:

Here no. of rows = no. of columns

∴ The given pbm is balanced

Step : 1

Row Difference

$$\begin{pmatrix} 0 & 2 & 7 & 12 & 5 \\ 0 & 3 & 11 & 14 & 4 \\ 0 & 4 & 12 & 12 & 4 \\ 0 & 0 & 3 & 5 & 1 \\ 0 & 0 & 5 & 15 & 0 \end{pmatrix}$$

Step : 2

Column Difference

$$\begin{pmatrix} \boxed{0} & 2 & 4 & 7 & 5 \\ \cancel{0} & 3 & 8 & 9 & 4 \\ \cancel{0} & 4 & 9 & 7 & 4 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & 1 \\ \cancel{0} & \cancel{0} & 2 & 10 & \boxed{0} \end{pmatrix}$$

Step : 3

$$\begin{pmatrix} \cancel{0} & \boxed{0} & 2 & 5 & 3 \\ \boxed{0} & 1 & 6 & 7 & 2 \\ \cancel{0} & 2 & 7 & 5 & 2 \\ 2 & \cancel{0} & \boxed{0} & \cancel{0} & 1 \\ 2 & \cancel{0} & 2 & 10 & \boxed{0} \end{pmatrix}$$

Step : 4

$$\begin{pmatrix} 1 & \boxed{0} & 2 & 5 & 3 \\ \boxed{0} & \cancel{0} & 5 & 6 & 1 \\ \cancel{0} & 1 & 6 & 4 & 1 \\ 3 & \cancel{0} & \boxed{0} & \cancel{0} & 1 \\ 3 & \cancel{0} & 2 & 10 & \boxed{0} \end{pmatrix}$$

Step : 5

$$\begin{pmatrix} 2 & \boxed{0} & 2 & 5 & 3 \\ \boxed{0} & \cancel{0} & 4 & 5 & \cancel{0} \\ \cancel{0} & \cancel{0} & 5 & 3 & \cancel{0} \\ 4 & \cancel{0} & \boxed{0} & \cancel{0} & 1 \\ 4 & \cancel{0} & 2 & 10 & \boxed{0} \end{pmatrix}$$

Step : 6

$$\begin{pmatrix} 2 & \boxed{0} & \cancel{0} & 3 & 3 \\ \boxed{0} & \cancel{0} & 2 & 3 & \cancel{0} \\ \cancel{0} & \cancel{0} & 3 & 1 & \boxed{0} \\ 6 & \cancel{0} & \cancel{0} & \boxed{0} & 1 \\ 4 & \cancel{0} & \boxed{0} & 8 & \cancel{0} \end{pmatrix}$$

Here each row and column having only one assignment.

Assignment schedule is,

$A \to W$, $B \to V$, $C \to Z$, $D \to Y$, $E \to X$.

Min time $= 5 + 4 + 12 + 10 + 15 = 46$ days.

H.W

1. Solve the assignment problem

$$\begin{array}{c|c} & \begin{matrix} \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{bmatrix} \end{array}$$

Here no. of rows = no of columns

$\therefore$ The given problem is balanced.

Step 1: Row Difference

$$\begin{pmatrix} 7 & 3 & 1 & 5 & \boxed{0} \\ \boxed{0} & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & \boxed{0} & 4 \\ 4 & 3 & 1 & \cancel{0} & 3 \\ 4 & \boxed{0} & 8 & 4 & \cancel{0} \end{pmatrix}$$

step 2: column difference

$$\begin{pmatrix} 7 & 3 & \boxed{0} & 5 & 0 \\ \boxed{0} & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & \boxed{0} & 4 \\ 4 & 3 & \boxed{0} & \cancel{0} & 3 \\ 4 & \boxed{0} & 7 & 4 & \cancel{0} \end{pmatrix}$$

step 3:

$$\begin{pmatrix} 11 & 3 & \cancel{0} & 9 & \boxed{0} \\ \boxed{0} & 5 & \cancel{0} & 5 & \cancel{0} \\ 1 & 2 & 2 & \boxed{0} & \cancel{0} \\ 8 & 3 & \boxed{0} & 4 & 3 \\ 8 & \boxed{0} & 1 & 8 & \cancel{0} \end{pmatrix}$$

Here each row and column having only one assignment

Assignmen schedule is

$A \to \text{V}$, $B \to \text{I}$, $C \to \text{IV}$, $D \to \text{III}$, $E \to \text{II}$

Min time : $1 + 0 + 2 + 1 + 5 = 9$ days

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2. Solve the assignment problem :

$$\begin{array}{c|c} & \begin{matrix} M_1 & M_2 & M_3 & M_4 \end{matrix} \\ \begin{matrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \end{matrix} & \begin{pmatrix} 9 & 22 & 58 & 11 \\ 43 & 78 & 72 & 50 \\ 41 & 28 & 91 & 37 \\ 74 & 42 & 27 & 49 \\ 36 & 11 & 57 & 22 \end{pmatrix} \end{array}$$

Ans:

Here no of rows $\neq$ no of columns

So adding dummy column M_5.

Step 1: Row difference

Adding dummy column

$$\begin{pmatrix} 9 & 22 & 58 & 11 & 0 \\ 43 & 78 & 72 & 50 & 0 \\ 41 & 28 & 91 & 37 & 0 \\ 74 & 42 & 27 & 49 & 0 \\ 36 & 11 & 57 & 22 & 0 \end{pmatrix}$$

Step 1: Row difference

$$\begin{pmatrix} 9 & 22 & 58 & 11 & 0 \\ 43 & 78 & 72 & 50 & 0 \\ 41 & 28 & 91 & 37 & 0 \\ 74 & 42 & 27 & 49 & 0 \\ 36 & 11 & 57 & 22 & 0 \end{pmatrix}$$

Step 2: column difference

$$\begin{pmatrix} \boxed{0} & 11 & 31 & \cancel{0} & \cancel{0} \\ 34 & 67 & 45 & 39 & \boxed{0} \\ 32 & 17 & 64 & 26 & \cancel{0} \\ 65 & 31 & \boxed{0} & 38 & \cancel{0} \\ 27 & \boxed{0} & 30 & 11 & \cancel{0} \end{pmatrix}$$

Step 3:

$$\begin{pmatrix} \boxed{0} & 22 & 42 & \cancel{0} & 11 \\ 23 & 67 & 45 & 28 & \boxed{0} \\ 21 & 17 & 64 & 15 & \cancel{0} \\ 54 & 31 & \boxed{0} & 27 & \cancel{0} \\ 16 & \boxed{0} & 30 & \cancel{0} & \cancel{0} \end{pmatrix}$$

Step 4:

$$\begin{pmatrix} \boxed{0} & 22 & 57 & \cancel{0} & 26 \\ 8 & 52 & 45 & 13 & \boxed{0} \\ 6 & 2 & 64 & \boxed{0} & \cancel{0} \\ 39 & 16 & \boxed{0} & 12 & \cancel{0} \\ 16 & \boxed{0} & 45 & \cancel{0} & 15 \end{pmatrix}$$

Here each row and column having only one assignment

Assignment schedule is

$J_1 \to M_1$, $J_2 \to M_5$, $J_3 \to M_4$, $J_4 \to M_3$, $J_5 \to M_2$

Min time $= 9+0+37+27+11 = 84$ days.

2. Balanced assignment problem:

A assignment problem is said to be Balanced if the number of rows is equal to the numbers of columns.

Number of rows = Number of columns.

3) unbalanced assignment problem:

An assignment problem is said to be unbalanced if the number of rows is not equal to the number of columns.

Number of rows $\neq$ Number of columns.

4) Different Between the transportation problem and the Assignment problem.

Transportation problem	Assignment problem
1. supply at any source	supply at any source

may be any positive Quantity a_i	(machine) will be 1. (ie) $a_i = 1$
2. Demand at any destination may be any positive quantity b_j	2. Demand at any destination (job) will be 1 (ie). $b_j = 1$
3. one (or) more source to any number of destinations.	one sources (machine) only one destination job.