7.3. Laws of Transverse Vibration of Strings

There are three laws of transverse vibration of strings :

(1) The fundamental frequency is inversely proportional to the length of the string

$$n \propto \frac{1}{l}$$

(2) The fundamental frequency is directly proportional to the square root of the stretching force or tension

$$n \propto \sqrt{T}$$

(3) The fundamental frequency is inversely proportional to the square root of the mass per unit length

$$n \propto \frac{1}{\sqrt{m}}$$

Combining the above three laws,

$$n \propto \frac{1}{l}\sqrt{\frac{T}{m}} \quad \text{or} \quad n = \frac{k}{l}\sqrt{\frac{T}{m}}$$

The value of the constant $k = \frac{1}{2}$

$$\therefore \qquad n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \ldots(1)$$

If D is the diameter of the wire and d is the density of the material of the wire, then

$$m = \frac{\pi D^2}{4} \times 1 \times d$$

$$n = \frac{1}{2l}\sqrt{\frac{4T}{\pi D^2 d}}$$

$$n = \frac{1}{lD}\sqrt{\frac{T}{\pi d}} \qquad \ldots(2)$$

7.4. Verification of the Laws of Transverse Vibration of Strings

(1) $n \propto \frac{1}{l}$ or. nl = constant

Take a sonometer and a tuning fork of frequency 256 (Fig. 7.3). Keeping a load of 1 kg find the position of the two bridges when the wire is in unsion with

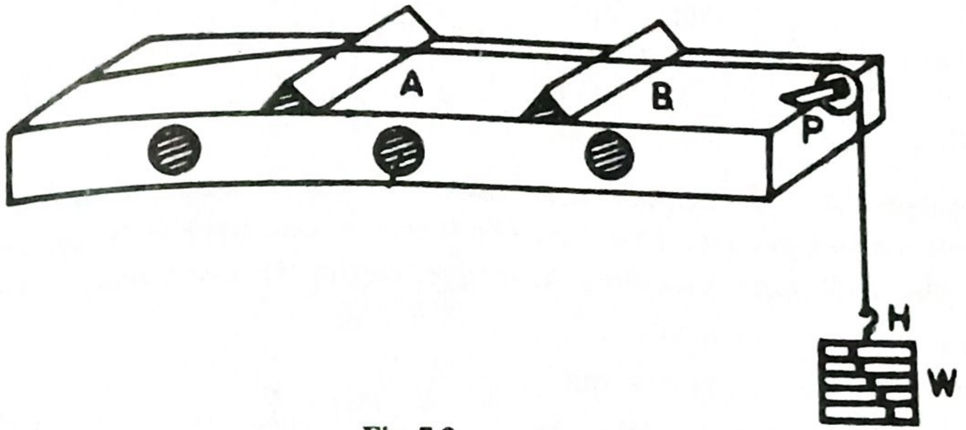

Fig. 7.3.

the tuning fork. This can be found by using a small paper rider on the sonometer wire. Suppose the length = l_1. Now, using a tuning fork of frequency 512, find the reasonating length, keeping tension the same. Let this length be l_2. It is found that l_2 is half l_1. Also $n_1 l_1 = n_2 l_2$.

$$n \propto \frac{1}{l} \quad \text{or} \quad nl = \text{constant}$$

(2)
$$n \propto \sqrt{T}$$

For a tuning fork of frequency 256, find the resonating length l_1 for a load of 1 kg. If will be found that when the load is increased to 4 kg the tuning fork of frequency 512 is in resonance with the same length of the wire l_1. Here

$$\frac{n_2}{n_1} \sqrt{\frac{T_2}{T_1,}}, \quad \text{or } n \propto \sqrt{T}$$

Also $n^2 \propto T$ or $T \propto n^2$

(3) $n \propto \frac{1}{\sqrt{m}}$

Take two wires of the same material and of diameters in the ratio of 1 : 2. Stretch the first wire with a tension of 1 kg wt. Find the resonating length with a tuning fork of frequency 512. Now stretch the second wire with the same tension of 1 kg and find the resonating length with a tuning fork of frequency 256. It is found that the two lengths are equal.

Here $$\frac{n_1}{n_2} = \frac{D_2}{D_1}$$

$$m_1 = \frac{\pi D_1^2 \times d}{4} \quad \text{and} \quad m_2 = \frac{\pi D_2^2 \times d}{4}$$

But $$\frac{m_2}{m_1} = \frac{D_2^2}{D_1^2} \quad \text{or} \quad \frac{D_2}{D_1} = \sqrt{\frac{m_2}{m_1}}$$

∴ $$\frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}} \quad \text{or} \quad n \propto \frac{1}{\sqrt{m}}.$$

17.2. A.C. FREQUENCY MEASUREMENT USING SONOMETER

The frequency of the alternating current mains in the laboratory can be determined using a sonometer.

Description : A sonometer consists of a thin uniform wire stretched over two bridges on a wooden box (Fig.17.4). One end of the wire is fixed to a peg. The other end of the wire passes over a pulley and carries a weight hanger. The length of the vibrating segment of the wire can be altered with the help of the movable bridges. The length of the vibrating segment can be measured by a scale fixed below the wire.

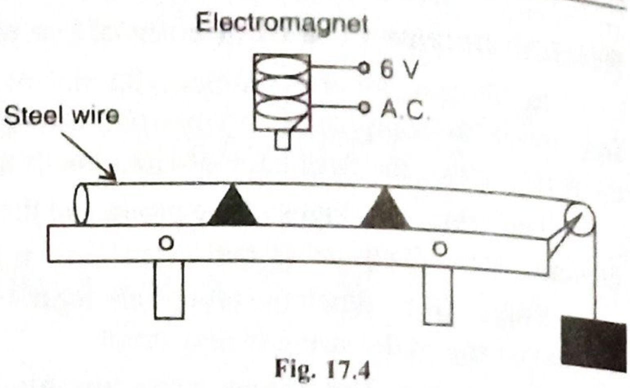

Fig. 17.4

Experiment : A steel wire is mounted on a sonometer under suitable tension. An electromagnet is excited by the low voltage alternating current whose frequency is to be determined. The electromagnet is placed just above the sonometer wire. The wire is attracted twice in each cycle.

A small paper rider is placed on the wire. The length of the wire is adjusted until the paper rider placed at the centre of the vibrating segment is thrown off. The length of the vibrating segment (l) is measured. The experiment is repeated for different tensions. The readings are tabulated as shown below:

No.	*Tension* (T)	*Length of the vibrating segment* l	$\frac{\sqrt{T}}{l}$

The mean value of $\frac{\sqrt{T}}{l}$ is found.

The mass per unit length of the wire is determined by finding the mass of a given length of the wire.

Calculation : The frequency of sonometer wire is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}.$$

During both the positive peak and the negative peak of the A.C., the wire is pulled by the electromagnet. So the wire vibrates twice, for each cycle of the A.C.

The frequency of the A.C. supply is given by

$$f = \frac{n}{2}.$$

Hence, the frequency of A.C. mains is calculated.

For brass or Copper wire

The ends of the secondary of a transformer are connected to the two ends A and B of the wire (Fig. 17.5). The P.D. at the ends of the transformer should be about 6 volts. The wire is set between the poles of a powerful horse shoe magnet or the opposite poles of two equal bar magnets, so that the magnetic field is in a horizontal plane and at right angles to the length of the wire.

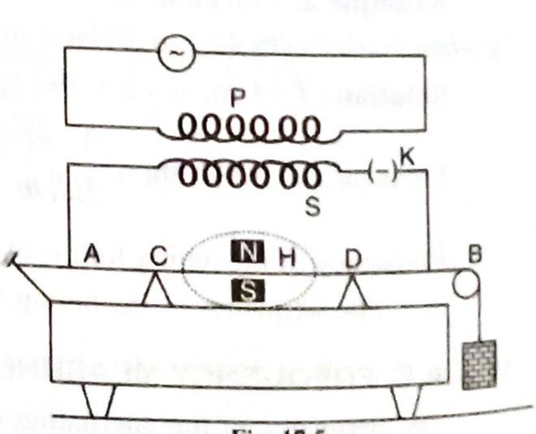

Fig. 17.5

The sonometer wire is subjected to a load of 150 gm. The wire is thrown into clear vibrations. The length of the wire is adjusted by moving the bridges C and D, till a single loop is seen between them with maximum amplitude. The distance CD is measured with a metre scale. The experiment is repeated for loads of 200, 250, 300, 350, 400,450 and 500 gm.

The length CD is measured in each case.

The frequency of the wire $n = \frac{1}{2l}\sqrt{\frac{Mg}{m}}$.

In this arrangement, the frequency of the wire is equal to the frequency of the A.C. mains.

ULTRASONICS

11.9. INTRODUCTION

The human ear is sensitive to sound waves in the frequency range from 20 to 20,000 Hz. This range is called audible range. Sound waves of frequency more than 20,000 Hz are called *ultrasonics*. These frequencies are beyond the audible limit.

These waves also travel with the speed of sound (330 ms^{-1}).

These waves exhibit the properties of audible sound waves and also show some new phenomena.

Their wavelengths are small.

11.10. PIEZOELECTRIC EFFECT

If one pair of opposite faces of a quartz crystal is subjected to pressure, the other pair of opposite faces develops equal and opposite electric charges on them (Fig. 11.11). The sign of the charges is reversed when the faces are subjected to tension instead of pressure. The electric charge developed is proportional to the amount of pressure or tension. This phenomenon is called *Piezoelectric effect.*

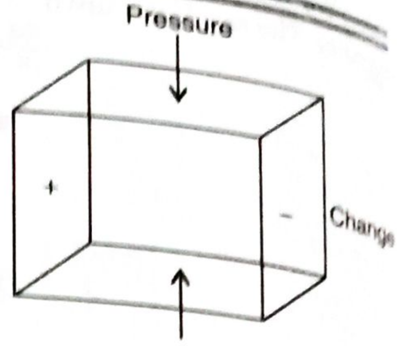

Fig. 11.11

The effect is *reversible, i.e.*, if an electric field is applied across one pair of faces of the crystal, contraction or expansion occurs across the other pair.

When the two opposite faces of a quartz crystal, their faces being cut perpendicular to the optic axis, are subjected to alternating voltage, the other pair of opposite faces experiences stresses and strains. The quartz crystal will continuously contract and expand. Elastic vibrations are set up in the crystal.

When the frequency of the alternating voltage is equal to the natural frequency of vibration of the crystal or its simple higher multiples, the crystal is thrown into resonant vibrations. The amplitude is large. These vibrations are longitudinal in nature.

Consider a *X*-cut crystal plate of thickness *t*. The fundamental frequency of vibration is given by

$$n = \frac{1}{2t}\sqrt{\frac{E}{\rho}}.$$

E is the Young's modulus and ρ is the density of the material of the crystal plate.

Example 1. *A quartz crystal of thickness 0.001 m is vibrating at resonance. Calculate the fundamental frequency. Given E for quartz* $= 7.9 \times 10^{10}$ Nm^{-2} *and* ρ *for quartz* $= 2650$ $kg\ m^{-3}$.

Solution.
$$n = \frac{1}{2t}\sqrt{\frac{E}{\rho}} = \frac{1}{2 \times 0.001}\sqrt{\frac{(7.9 \times 10^{10})}{2650}} = \mathbf{2.73 \times 10^6\ Hz.}$$

Example 2. *A piezoelectric X-cut quartz plate has a thickness of 1.5 mm. If the velocity of propagation of longitudinal sound waves along the X direction is 5760 m/s, calculate the fundamental frequency of the crystal.*

Solution. For the fundamental mode of vibration,

$$\text{thickness} = \frac{\lambda}{2}$$

$$\therefore \quad \lambda = 2 \times \text{thickness} = 2 \times (1.5 \times 10^{-3})\ \text{m} = 3 \times 10^{-3}\text{m}$$

Frequency,
$$n = \frac{v}{\lambda} = \frac{5760}{(3 \times 10^{-3})} = \mathbf{1.92 \times 10^6\ Hz}$$

11.11. PRODUCTION OF ULTRASONIC WAVES — PIEZOELECTRIC CRYSTAL METHOD

Principle. This is based on the inverse piezoelectric effect. When a quartz crystal is subjected to an alternating potential difference along the electric axis, the crystal is set into elastic vibrations along its mechanical axis. If the frequency of the electric oscillations coincides with the natural frequency of the crystal, the vibrations will be of large amplitude. If the frequency of the electric field is in the ultrasonic frequency range, the crystal produces ultrasonic waves.

Construction. The circuit diagram is shown in Fig. 11.12.

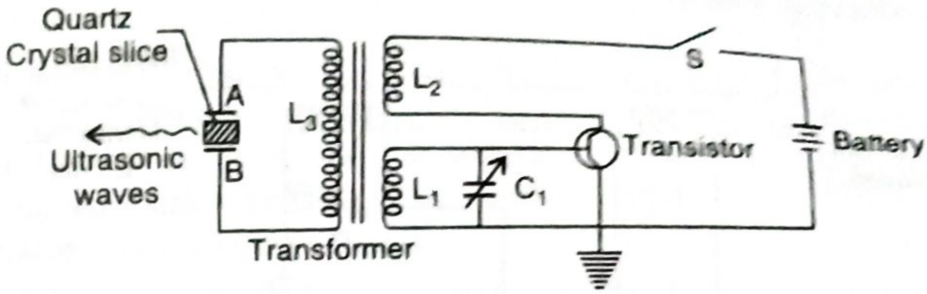

Fig. 11.12

It is a base tuned oscillator circuit. A slice of quartz crystal is placed between the metal plates A and B so as to form a parallel plate capacitor with the crystal as the dielectric. This is coupled to the electronic oscillator through primary coil L_3 of the transformer.

Coils L_2 and L_1 of oscillator circuit are taken from the secondary of the transformer. The collector coil L_2 is inductively coupled to base coil L_1. The coil L_1 and variable capacitor C_1 form the tank circuit of the oscillator.

Working. When the battery is switched on, the oscillator produces high frequency oscillations. An oscillatory e.m.f. is induced in the coil L_3 due to transformer action. So the crystal is now under high frequency alternating voltage.

The capacitance of C_1 is varied so that the frequency of oscillations produced is in resonance with the natural frequency of the crystal. Now the crystal vibrates with large amplitude due to resonance. Thus high power ultrasonic waves are produced.

Advantages

1. Ultrasonic frequencies as high as 500 MHz can be generated.
2. The output power is very high. It is not affected by temperature and humidity.
3. It is more efficient than magnetostriction oscillator.
4. The breadth of the resonance curve is very small. So we can get a stable and constant frequency of ultrasonic waves.

Disadvantages

1. The cost of the quartz crystal is very high.
2. Cutting and shaping the crystal is very complex.

heaps at the nodes.

11.14. PROPERTIES OF ULTRASONIC WAVES

1. They have a high energy content.
2. Just like ordinary sound waves, ultrasonic waves get reflected, refracted and absorbed.
3. They show negligible diffraction due to their small wavelength. Hence they can be transmitted over long distances without any appreciable loss of energy.
4. They produce intense heating effect when passed through a substance.
5. If an arrangement is made to form stationary waves of ultrasounds in a liquid, it serves as a diffraction grating. It is called an acoustic grating.

11.16. APPLICATIONS OF ULTRASONIC WAVES

(i) Science.

1. Investigation of structure of matter

We can determine the velocity of ultrasonics in liquids and gases and its variation with frequency and temperature. This study gives information about a number of properties of the medium such as its compressibility, absorption, concentration, specific heat capacity, chemical structure, arrangement of atoms in them etc.

2. Study of molecular energies

The frequencies of molecular vibrations are of the same order as the ultrasonic vibrations. So ultrasonic waves are used in the study of molecular energies. They are used in molecular acoustics for investigating structure and properties of substances.

3. Elastic symmetries of crystals

When ultrasonic waves are applied to certain crystals, they give rise to diffraction images. The diffraction images reveal the elastic symmetries of crystals.

(ii) Industrial Applications

1. Non-destructive testing (NDT)

Principle. *Whenever there is a change in medium, the ultrasonic waves will be reflected. Since the flaws can be detected without destroying the materials, it is called non-destructive testing.*

Working: The pulse echo system used to determine the various flaws like cracks, holes, air bubbles, laminations, etc., in the specimen is shown in Fig. 11.15.

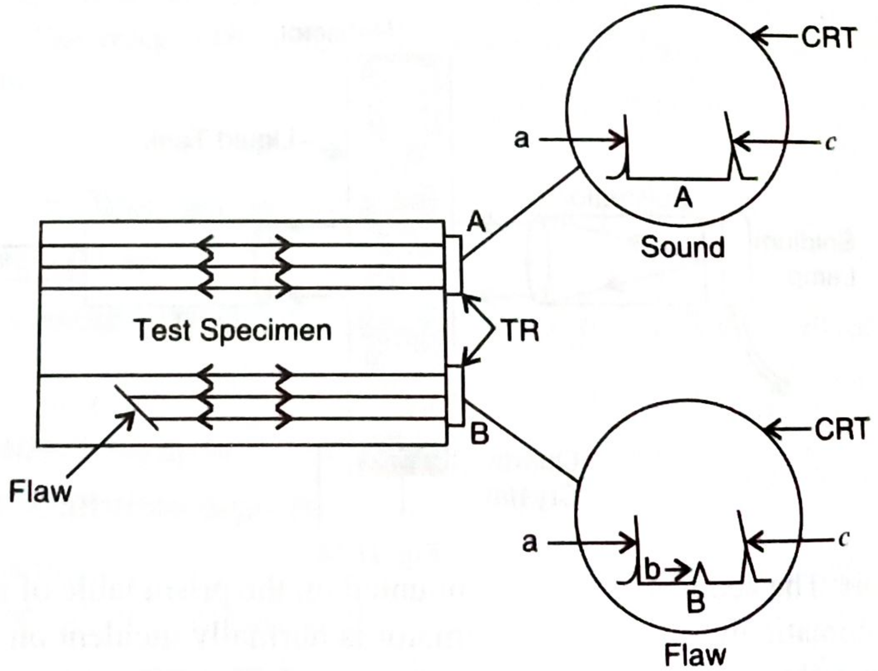

Fig. 11.15

a - Incident pulse

b - Pulse from flaw

c - Reflected pulse from the boundary of the specimen

TR - Transducer

CRT - Cathode Ray Tube.

Here short pulses of ultrasonic waves are transmitted into the material being tested. These pulses get reflected from discontinuities on their path or from any boundary of the material on which they strike. The received echoes are then displayed on a cathode ray tube screen. The CRT screen furnishes specific data as to the relative size of a discontinuity in terms of signal amplitude. The location of the discontinuity with respect to the scanning surface can be obtained by proper calibration of the CRT time base scale.

2. Ultrasonic soldering

Ultrasonic solders are used for soldering aluminium coil capacitors, aluminium wires and plates without using any fluxes.

An ultrasonic soldering iron consists of an ultrasonic generator having a tip fixed at its end. The tip is heated by an electrical heating element. The tip of the soldering iron melts solder on the aluminium. The ultrasonic vibrator removes the aluminium oxide layer. The solder thus gets fastened to the clear metal without any difficulty.

3. Ultrasonic welding

The properties of some metals change on heating. Therefore, they cannot be welded by electric or gas welding. In such cases, the metal sheets are welded together at room temperature using ultrasonic waves.

4. Ultrasonic drilling and cutting

Ultrasonics are used for making holes in very hard materials such as glass, diamond, gems and ceramics.

5. Ultrasonics in metallurgy

To irradiate molten metals which are in the process of cooling, so as to refine the grain size and to prevent the formation of cores and to release trapped gases, the ultrasonic waves are used.

6. Formation of alloys

The constituents of alloys, having widely different densities, can be mixed uniformly by a beam of ultrasonics. Thus it is easy to get alloys of uniform composition.

7. Acoustic holograms

Surface structures of various engineering materials used for space applications can be studied by using acoustic holograms.

8. Sound navigation and ranging (SONAR)

Ultrasonic waves sent from a point A travel through sea water and get reflected back from the bottom of the sea (Fig. 11.16). The reflected waves are received at the point B. Using a CRO, the time taken t, for the ultrasonic wave to travel to the bottom of the sea and reflected back to the surface is calculated.

Let v = velocity of ultrasonic wave in the sea water.

$$\text{Depth of the sea} = \frac{v \times t}{2}$$

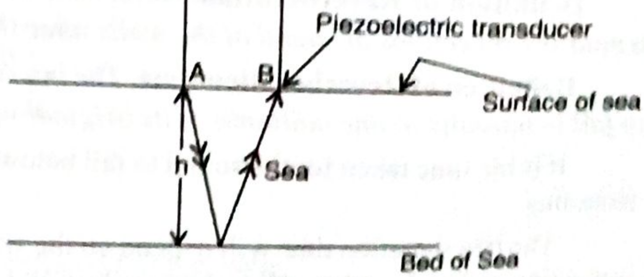

Fig. 11.16

The same method is used for finding the distance and direction of a submarine. The change in frequency of the echo signal due to Doppler effect helps to determine the velocity of the submarine and its direction. The whole system is called SONAR.

9. Chemical applications

1. Ultrasonic waves are used to form stable emulsions of even immiscible liquids like *water and oil* or *water and mercury*. This finds an application in the preparation of photographic films, face creams etc.
2. They are used to liquefy gels like aluminium hydroxide in the same way as they are liquefied by shaking.
3. They are used to coagulate fine solid or liquid particles in a gas ; for example, dust, smoke, mist etc. Ultrasonics thus find use in collecting factory dust and purifying the air.
4. Ultrasonics act like a catalytic agent and accelerate chemical reactions. Ultrasonic waves accelerate crystallisation.

(*iii*) Medical Applications

1. Disease treatment. Ultrasonic therapy can be used to treat diseases like bursites, abscesses, neuralgic and rheumatic pains etc.

2. Surgical use. Kidney stones and brain tumours can be removed without shedding any blood using ultrasonic waves. Also, any tissue in our body can be cut selectively during an operation using ultrasonics.

3. Diagnostic use. Ultrasonics are used for detecting tumours and other defects in human body. State of breast cancer can be identified using ultrasonics in a non-destructive manner. Also, the twins or any defect in the growth of foetus can be identified using ultrasonics before delivery.

4. Extraction of broken teeth. Dentists use ultrasonic waves to properly extract broken teeth.

5. Sterilization. Ultrasonic waves can kill bacteria. Therefore, they are used for sterilising milk.

6. Blood flow meters. Ultrasonic Doppler blood flow meters are used to study the blood flow velocities in blood vessels of our body.

Acoustics of buildings

The subject of physics which deals with the design and construction of rooms or halls so as to give the best sound effects is called acoustics of buildings or architectural acoustics.

The acoustical properties of a room or hall have considerable effect on the clarity and intelligibility of speech or music produced in the hall.

2.4 ABSORPTION COEFFICIENT

When a sound wave strikes a surface, a part of its energy is absorbed, a part of it is transmitted and the remaining part is reflected.

The property of the surface to convert sound energy into other forms of energy is known as absorption.

The effectiveness of absorption of sound energy by the surface is expressed as absorption coefficient.

Absorption coefficient (a) is defined as the ratio of sound energy absorbed by its surface to that of total sound energy incident on the surface.

$$a = \frac{\text{Sound energy absorbed by the surface}}{\text{Total sound energy incident on the surface}}$$

Practical definition of absorption coefficient

In order to compare the relative sound absorption of different materials the open window is taken as standard reference since it is a perfect sound absorber.

It is so because the whole of the sound energy passes through the open window and none is reflected.

***Absorption coefficient of a surface is the ratio of sound energy absorbed by* 1 m^2** of the surface to that absorbed by **1 m^2** of an open window.

$$a = \frac{\text{Sound energy absorbed by } 1\text{m}^2 \text{ of the surface}}{\text{Sound energy absorbed by } 1\text{m}^2 \text{ of open window}}$$

Reverberation

The sound which is produced in a hall, travels in all directions and undergo multiple reflections from the walls, floor and ceiling before it becomes inaudible (fig. 2.3).

A listener in the room continuesly receives successive reflections of diminishing intensity of sound (a part of sound energy is lost at each reflection). Therefore, the listener hears a ***'roll of sound'*** instead of a single sharp sound.

This implies that the sound is heard continuously for short definite time interval even after the source of sound has stopped to emit the sound.

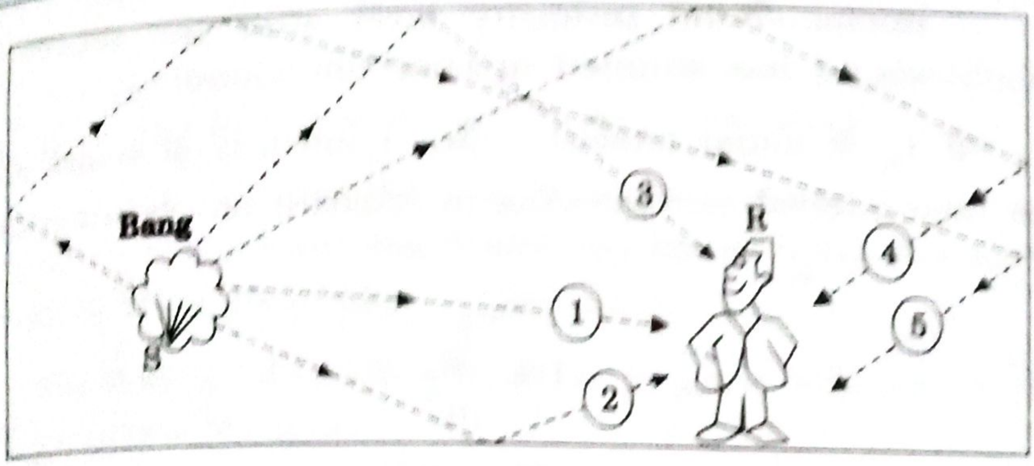

Fig. 2.3 Reverberation of sound in a room

The existence or prolongation or persistence of sound in a room (due to multiple reflections from surfaces) even after the source of sound has stopped to emit the sound is called reverberation.

This familiar phenomenon is experienced in vacant halls of a new building.

Reverberation Time

Definition

- **The time duration for which a sound persists even after the source of sound has stopped to emit the sound is called reverberation time.**
- **It is measured as the time taken by the sound to fall below the minimum audibility after source of sound has stopped to emit the sound.**

Standard reverberation time

- **Standard reverberation time is defined as time taken by the sound intensity to fall one - millionth (10^{-6}) of its initial intensity after the sound source has stopped to emit the sound.**
- **It is also defined as time taken by the sound intensity level to reduce by 60 decibels from its**

initial sound intensity level after the source of sound has stopped to emit the sound.

If I_m is initial intensity, then I intensity of sound after the time interval corresponding to standard reverberation time 'T'

$$\text{ie., } \quad I = \frac{I_m}{10^6}$$

$$\boxed{\frac{I}{I_m} = 10^{-6}}$$

2.6 SABINE'S FORMULA FOR REVERBERATION TIME

Sabine derived a relation for the standard reverberation time.

It is given by $\boxed{T = \dfrac{0.167\, V}{\Sigma\, as}}$ second

$$T = \frac{0.167\, V}{a_1 s_1 + a_2 s_2 + \dots}$$

where V – Volume of the room or hall in $\mathbf{m^3}$

a – Absorption coefficients of surface areas of different materials present in the hall in **O.W.U.**

s – Surface areas of the different surfaces in $\mathbf{m^2}$

$\Sigma\, as$ – Total absorption of sound i.e., sum of the product of absorption coefficients and surface areas of the different surfaces present in the hall in **O.W.U.** m^2 **or sabine**

It is popularly known as ***Sabine's formula for reverberation time***.

2.7 DERIVATION USING GROWTH AND DECAY METHOD

Let us assume that the sound energy is uniformly distributed throughout the hall. It does not depend on frequency.

We shall calculate the rate at which the sound energy is incident upon the walls and hence the rate at which the sound energy is being absorbed.

Consider a small element ds on a plane wall AB in the hall as shown in fig. 2.4.

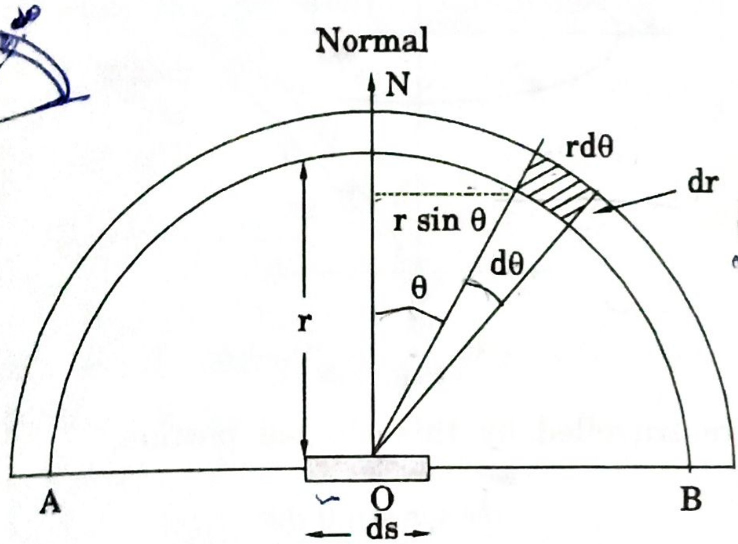

Fig. 2.4 Sound absorption on a plane wall

It is assumed that the element ds receives sound energy. Taking O as a mid point on ds, two semicircles are drawn with radii r and $r + dr$.

Now, consider a small shaded portion between the circles lying between two radii r and $r + dr$ drawn at angles θ and $\theta + d\theta$ with normal ON as shown in fig. 2.4.(a).

Radial length of the shaded portion $= dr$

Arc length of the shaded portion $= r\, d\theta$

Area of this shaded portion $= r\, d\theta\, dr$ ...(1)

Imagine, the whole figure is rotated about the normal through an angle $d\phi$ (radius of the rotating shaded portion being $r \sin\theta$).

The shaded portion travels through a small distance d_x (circumferential length) and thus, traces out an elemental volume dV (Fig. 2.5).

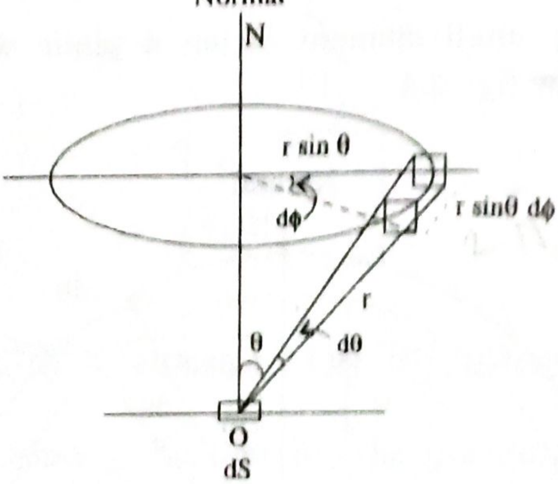

Fig. 2.5

Distance travelled by this shaded portion,

$$dx = r \sin\theta \, d\phi$$

∴ Volume traced by the shaded portion,

$$dV = \text{area} \times \text{distance travelled}$$

$$dV = r\,d\theta\,dr \times r \sin\theta\,d\phi$$

$$dV = r^2 \sin\theta\,d\theta\,dr\,d\phi \quad \text{...(2)}$$

If E is the sound energy density i.e., sound energy per unit volume, then,

Sound energy present within the elemental volume dV

$$= E \times dV$$

On substituting eqn (2), we have

$$= Er^2 \sin\theta\,dr\,d\theta\,d\phi \quad \text{...(3)}$$

This sound energy from elemental volume is travelling equally in all directions in total solid angle of 4π.

∴ Sound energy travels the volume dV per unit solid angle

$$= \frac{E\,dV}{4\pi} = \frac{Er^2 \sin\theta\, dr\, d\theta\, d\phi}{4\pi} \quad ...(4)$$

In this case, the solid angle subtended by the area ds at this elemental volume dV

$$= \frac{ds \cos\theta}{r^2}$$

Hence, sound energy from the elemental volume dV towards 'ds' is given by

$$= \frac{Er^2 \sin\theta\, d\theta\, dr\, d\phi}{4\pi} \times \frac{ds\cos\theta}{r^2}$$

$$= \frac{E\,ds}{4\pi} \sin\theta \cos\theta\, d\theta\, d\phi\, dr \quad ...(5)$$

Since sound energy is falling on ds from all directions, θ changes from 0 to $\pi/2$ and ϕ changes from 0 to 2π.

Further, to get total sound energy received per second, r changes from 0 to v, where v is the velocity of sound (since sound existing within the distance of 0 to v metre from ds reaches ds in one second).

Note:

Solid angle is an angle formed at the vertex of a cone or subtended at the point of intersection of two or more planes. (Fig. 2.6)

Solid angle subtended by an area *ds* about any point *P* at a distance '*r*' is given by

$$d\omega = \frac{\text{Normal component of area}}{\text{Square of the distance}} = \frac{ds \cos\theta}{r^2}$$

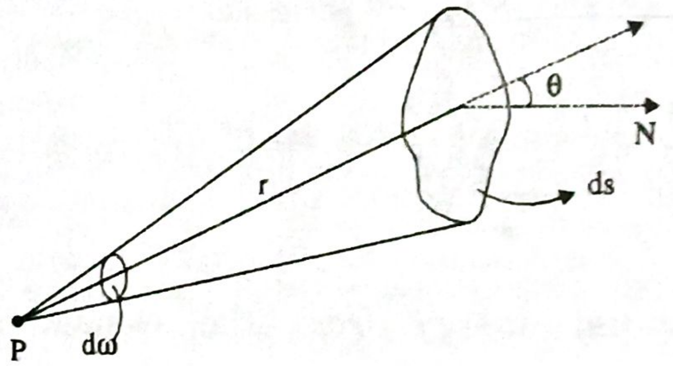

Fig. 2.6 Solid angle

where θ is angle between the normal to the area *ds* and *r* (line joining the point *P* and the surface *ds*)

Total solid angle subtended by the sphere of radius *r* is given by

$$\omega = \frac{\text{Surface area of the sphere}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Unit for solid angle is steradian (Sr).

∴ Total sound energy falling on *ds* per second

$$= \frac{Eds}{4\pi} \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\phi \int_0^{v} dr$$

$$= \frac{E\,ds}{4\pi} \times \frac{1}{2} \times 2\pi \times v$$

$$= \frac{Ev\,ds}{4} \qquad ...(6)$$

If a is the absorption coefficient of the wall AB of which ds is a part, then sound energy absorbed by ds in one second

$$= \frac{Ev\,ds\,a}{4}$$

$\therefore$ Total sound energy absorbed per second by the whole enclosure (entire hall)

$$= \frac{Ev\,\Sigma\,a\,ds}{4}$$

$$= \frac{EvA}{4} \qquad ...(7)$$

where $A = \Sigma\,ads$ is total absorption of sound by all the surfaces inside the hall on which sound energy is incident.

Growth and decay of sound energy

If P is sound power output, i.e rate of emission of sound energy from the source and V is the total volume of the hall, then,

Total sound energy in the hall at a given instant 't' $= EV$

where E is the sound energy density at that instant.

$\therefore$ Rate of growth or increase in energy per second

$$= \frac{d\,(EV)}{dt} = V\frac{dE}{dt} \qquad ...(8)$$

[$\because$ Volume of the hall (V) is constant.]

Rate of emission of sound energy by the source	=	Rate of growth of sound energy in the room	+	Rate of absorption of sound energy by the walls

$$\text{ie.,} \quad P = V\frac{dE}{dt} + \frac{EvA}{4} \quad \ldots(9)$$

When steady state is reached, $\frac{dE}{dt} = 0$,

Steady-state energy density is denoted as E_m, and it is expressed as

$$P = \frac{E_m v A}{4}$$

$$\therefore \quad \boxed{E_m = \frac{4P}{vA}}$$

Dividing equation (9) by V, we have

$$\frac{dE}{dt} + \frac{EvA}{4V} = \frac{P}{V} \quad \ldots(10)$$

Substituting $\frac{vA}{4V} = \alpha$, eqn (10) is written as

$$\frac{dE}{dt} + E\alpha = \frac{4P\alpha}{vA} \qquad \left(\because \frac{1}{V} = \frac{4\alpha}{vA}\right)$$

Multiplying with $e^{\alpha t}$ on both sides of the equation, we get

$$\left[\frac{dE}{dt} + E\alpha\right]e^{\alpha t} = \frac{4P\alpha\, e^{\alpha t}}{vA}$$

$$\frac{d}{dt}(Ee^{\alpha t}) = \frac{4P\alpha\, e^{\alpha t}}{vA}$$

Integrating on both sides, we obtain

$$\int \frac{d}{dt}(Ee^{\alpha t}) = \int \frac{4P\alpha e^{\alpha t}}{\upsilon A} = \frac{4P\alpha}{\upsilon A}\int e^{\alpha t} \quad \left[\because \int e^{\alpha t} = \frac{e^{\alpha t}}{\alpha}\right]$$

$$Ee^{\alpha t} = \frac{4P\alpha e^{\alpha t}}{\upsilon A \alpha} + K$$

$$\boxed{Ee^{\alpha t} = \frac{4P e^{\alpha t}}{\upsilon A} + K} \quad ...(11)$$

where K is a constant of integration. The value of K is determined by considering the boundary conditions.

Growth of Sound Energy

Sound energy grows from the instant the source begins to emit sound at $t = 0$ and $E = 0$

Applying this condition to equation (11), we get

$$0\,e^{0} = \frac{4P e^{0}}{\upsilon A} + K$$

$$\therefore \quad K = \frac{-4P}{\upsilon A} \quad ...(12)$$

$$\left[\because e^{0} = 1\right]$$

Using eqn (12) in eqn (11), we get

$$Ee^{\alpha t} = \frac{4P}{\upsilon A}e^{\alpha t} - \frac{4P}{\upsilon A}$$

$$E = \frac{4P e^{\alpha t}}{\upsilon A\, e^{\alpha t}} - \frac{4P}{\upsilon A e^{\alpha t}}$$

$$E = \frac{4P}{\upsilon A} - \frac{4P}{\upsilon A}e^{-\alpha t}$$

$$E = \frac{4P}{vA}(1 - e^{-\alpha t})$$

$$\boxed{E = E_m (1 - e^{-\alpha t})} \quad \text{...(13)}$$

$\left[\because E_m = \frac{4P}{vA}\right.$ is the maximum sound energy density$\left.\right]$

The equation (13) represents the growth of sound energy density E with time t. A graph is plotted between sound energy density and time (t). It is a rising exponential curve as shown in fig. 2.7 This indicates that E increases with t, and when $t \rightarrow \infty$, $E = E_m$.

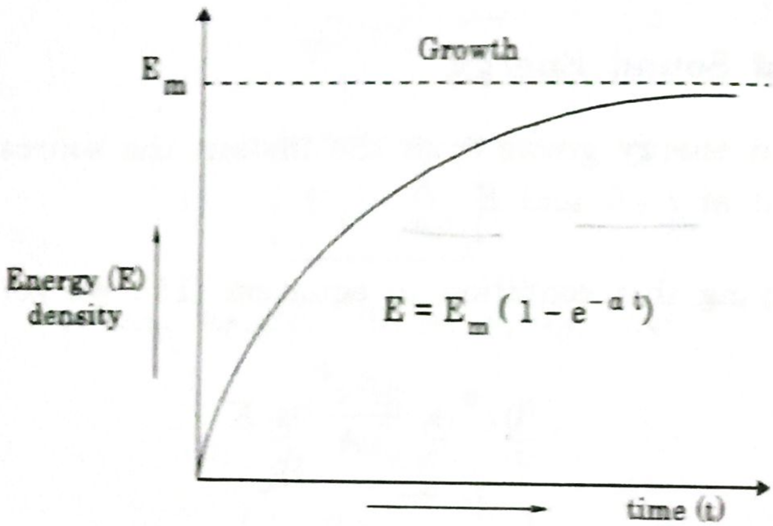

Fig. 2.7. Growth of sound with time

Decay of Sound Energy

Assume that when sound energy has reached its steady (maximum value) state E_m, source of sound is cut off.

Then, the rate of emission of sound energy

$$P = 0$$

Eqn (11) is written as $\quad Ee^{\alpha t} = K$

Substituting the boundary conditions

$E = E_m$ at $t = 0$ and $P = 0$ in equation (11), we get

$$E_m e^0 = 0 + K$$

$$K = E_m \quad ...(14)$$

From eqns (11) and (14), we get

$$Ee^{\alpha t} = E_m$$

$$E = \frac{E_m}{e^{\alpha t}} = E_m e^{-\alpha t}$$

$$\boxed{E = E_m e^{-\alpha t}} \quad ...(15)$$

Equation (15) represents the decay of sound energy density with time after the source is cut off. A graph is plotted between sound energy density and time. It is an exponentially decreasing curve as shown in fig. 2.8.

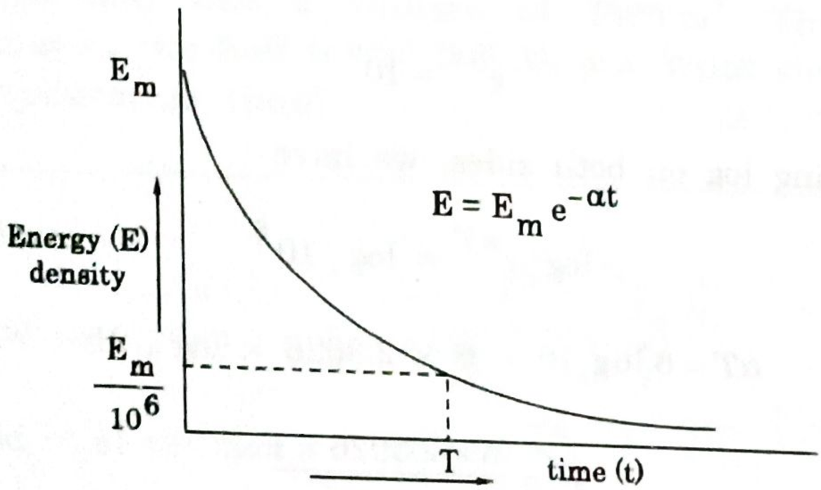

Fig. 2.8 Decay of sound with time

Expression for reverberation time

We know that standard reverberation time *T* is the time taken by the sound to fall of its intensity to one-millionth of its initial value after the source is cut off.

Now the sound energy density before cut off is E_m. At standard reverberation time, it reduces to

$$E = \frac{E_m}{10^6}$$

[Since E is proportional to I, $\frac{I}{I_m} = \frac{E}{E_m} = 10^{-6}$]

Hence, to calculate T,

we put $E = \frac{E_m}{10^6} = E_m \, 10^{-6}$

and $t = T$ in eqn (15)

$$E_m \, 10^{-6} = E_m \, e^{-\alpha T}$$

$$e^{-\alpha T} = 10^{-6}$$

$$e^{\alpha T} = 10^{6}$$

Taking log on both sides, we have

$$\log_e e^{\alpha T} = \log_e 10^6$$

$$\alpha T = 6 \log_e 10 = 6 \times 2.3026 \times \log_{10} 10$$

$$T = \frac{6 \times 2.3026 \times 1}{\alpha}$$

Substituting $\alpha = \frac{\upsilon A}{4V}$, we get

$$T = \frac{6 \times 2.3026 \times 1}{\frac{\upsilon A}{4V}}$$

$$= \frac{6 \times 2.3026 \times 4V}{\upsilon A}$$

Taking the velocity of the sound, $v = 330 \text{ ms}^{-1}$

we have, $$T = \frac{6 \times 2.3026 \times 4V}{330 \times A}$$

or $$T = \frac{0.167V}{A}$$

i.e. Reverberation Time = $$\boxed{T = \frac{0.167V}{\Sigma as}} \qquad (\because A = \Sigma as)$$

This equation is in agreement with the experimental values obtained by Sabine.

ANNA UNIVERSITY SOLVED PROBLEMS

Problem 2.3

A cinema hall has a volume of 7500 m^3. The total absorption in the hall is 825 O.W.U. m2. What should be the reverberation time? *[A.U. Dec 2010]*

Given data

Total absorption $\Sigma as = 825 \text{ O.W.U.m}^2$

Volume of the hall $V = 7500 \text{ m}^3$

Solution

Reverberation time $$T = \frac{0.167 V}{\Sigma as}$$

Substituting the given values, we have

$$= \frac{0.167 \times 7500}{825} = 1.52 \text{ second}$$