2.. CORRECTIONS TO POISEUILLE'S FORMULA

Two important corrections are to be applied in the Poiseuille's equation :

(*i*) Correction for pressure head : The outgoing liquid acquires K.E. due to its velocity after passing through the tube. Hence the pressure-head maintained is utilized not only for overcoming viscous resistance but also in imparting considerable K.E. to emergent liquid. So the effective pressure is less and is given by

$$p_1 = p - \frac{V^2 \rho}{\pi^2 a^4}.$$

This can be deduced as follows :

The K.E. given to the liquid of density ρ per second

$$E' = \int_0^a \frac{1}{2}(2\pi r\, dr v \rho)\, v^2 = \pi\rho \int_0^a r v^3\, dr$$

But

$$v = \frac{p}{4\eta l}(a^2 - r^2)$$

$$\therefore \quad E' = \pi\rho \int_0^a r \left(\frac{p}{4\eta l}\right)^3 (a^2 - r^2)^3\, dr$$

$$= \pi\rho \left(\frac{p}{4\eta l}\right)^3 \frac{a^8}{8} = \left(\frac{\pi p a^4}{8\eta l}\right)^3 \frac{\rho}{\pi^2 a^4}$$

$$= \frac{V^3 \rho}{\pi^2 a^4}$$

The work done in overcoming viscosity is $p_1 V$ whereas total work done per unit volume is pV. Here p_1 is the effective pressure.

$$\therefore \quad pV = p_1 V + \frac{V^3 \rho}{\pi^2 a^4}$$

or

$$p_1 = p - \frac{V^2 \rho}{\pi^2 a^4}$$

$$\therefore \quad p_1 = g\rho\left(h - \frac{V^2}{\pi^2 a^4 g}\right)$$

Thus $[V^2/(\pi^2 a^4 g)]$ is the correction factor to the pressure head for gain of kinetic energy by the emergent liquid.

(*ii*) Correction for length of tube: At the inlet end of the tube, the flow of the liquid is not streamline for some distance. Consequently the liquid is accelerated. The effective length of the tube is thus increased from l to $l + 1.64\,a$. Thus, the corrected relation for η becomes

$$\eta = \frac{\pi a^4}{8V(l + 1.64\,a)}\left(h - \frac{V^2}{\pi^2 a^4 g}\right) g\rho$$

2.3. POISEUILLE'S FORMULA FOR THE FLOW OF A LIQUID THROUGH A CAPILLARY TUBE

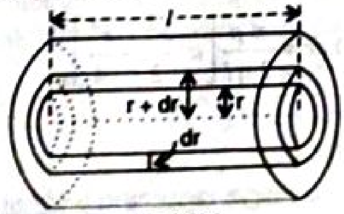


Fig. 2.3(a)

Fig. 2.3(b)

Suppose a constant pressure difference p is maintained between the two ends of the capillary tube of length l and radius a, as shown in Fig. 2.3 (a). Consider the steady flow of a liquid of coefficient of viscosity η through the tube. The velocity of the liquid is a maximum along the axis and is zero at the walls of the tube. Assume that there is no radial flow. Consider a cylindrical shell of the liquid co-axial with the tube of inner radius r and outer radius $r + dr$ [Fig. 2.3(b)]. Let the velocity of the liquid on the inner surface of the shell be v and that on the outer surface be $v - dv$. (dv/dr) is the velocity gradient.

The surface area of the shell $= A = 2\pi r l$.

According to Newton's law of viscous flow, the backward dragging tangential force exerted by the outer layer on the inner layer, opposite to the direction of motion

$$F_1 = -\eta A \frac{dv}{dr} = -\eta\, 2\pi r l \frac{dv}{dr}$$

The driving force on the liquid shell, accelerating it forward

$$F_2 = p\pi r^2$$

where, p = pressure difference across the two ends of the tube and

πr^2 = Area of cross-section of the inner cylinder.

When the motion is steady,

backward dragging force (F_1) = The driving force (F_2)

$$-\eta\, 2\pi r l \frac{dv}{dr} = p\pi r^2 \text{ or } dv = \frac{-p}{2\eta l} r\, dr.$$

Integrating, $$v = \frac{-p}{2\eta l}\frac{r^2}{2} + C.$$

where C is a constant of integration.

When $r = a$, $v = 0$. Hence, $0 = \frac{-p}{2\eta l}\frac{a^2}{2} + C$ or $C = \frac{pa^2}{4\eta l}$

$$\therefore \quad v = \frac{p}{4\eta l}(a^2 - r^2)$$

This gives us the average velocity of the liquid flowing through the cylindrical shell.

Hence the volume of the liquid that flows out per second through this shell

$$dV = \begin{pmatrix}\text{Area of cross-section of the shell}\\ \text{of radius } r \text{ and thickness } dr\end{pmatrix} \times \text{Velocity of flow}$$

$$= 2\pi r\, dr \frac{p}{4\eta l}(a^2 - r^2) = \frac{\pi p}{2\eta l}(a^2 r - r^3)\, dr$$

The volume of the liquid that flows out per second is obtained by integrating the expression for dV between the limits $r = 0$ to $r = a$.

$$V = \int_0^a \frac{\pi p}{2\eta l}(a^2 r - r^3)\, dr = \frac{\pi p}{2\eta l}\left[a^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^a$$

$$= \frac{\pi p}{2\eta l}\frac{a^4}{4}$$

or $$V = \frac{\pi p a^4}{8\eta l}$$

2.6 OSTWALD'S VISCOMETER

This instrument is used to compare the viscosities of two liquids. It is also used to study the variation of viscosity of a liquid with temperature.

The apparatus consists of two glass bulbs *A* and *B* joined by a capillary tube *DE* bent into a U-form (Fig. 2.5). The bulb *A* is connected to a funnel *F*. The bulb *B* is connected to an exhaust pump through a stop-cock *S*. *K*, *L*, and *M* are fixed marks, as shown in the figure. The whole apparatus is placed inside a constant temperature bath.

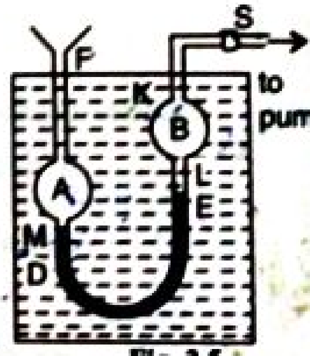


Fig. 2.5

The liquid is then introduced into the apparatus through the funnel and its volume is adjusted, so that the liquid occupies the portion between the marks *K* and *M*, when the stop-cock is closed. The stop-cock is now opened and with the help of the exhaust pump the liquid is sucked up above the mark *K*. The stop-cock is closed and the exhaust pump is removed. The stop-cock is again opened. The liquid is allowed to flow through the capillary tube.

The time (t_1) taken by the liquid to fall from the mark *K* to the mark *L* is noted. The experiment is then repeated with the second liquid and the time (t_2) taken by it to fall from *K* to *L* is noted.

Theory : Let η_1 and η_2 be the coefficients of viscosity and ρ_1 and ρ_2 the densities of the two liquids respectively. Let the volume of liquid between *K* and *L* be *V*. Then,

the rate of flow of the first liquid $= V_1 = V/t_1$ ...(1)

and the rate of flow of the second liquid $= V_2 = V/t_2$ ...(2)

Now, $\eta_1 = \dfrac{\pi . P_1 . a^4}{8 V_1 . l}$ and $\eta_2 = \dfrac{\pi . P_2 . a^4}{8 V_2 . l}$

or $$\frac{\eta_1}{\eta_2} = \frac{V_2}{V_1} \times \frac{P_1}{P_2} \quad ...(3)$$

But the pressure *P* is proportional to the density of the liquid used ($P = h\rho g$)

Hence, $$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \quad ...(4)$$

Also, dividing (2) by (1), $$\frac{V_2}{V_1} = \frac{t_1}{t_2} \quad ...(5)$$

Hence, $$\frac{\eta_1}{\eta_2} = \frac{t_1 . \rho_1}{t_2 . \rho_2} \quad ...(6)$$

From equation (6), η_1/η_2 can be calculated.

2.7 POISEUILLE'S METHOD FOR DETERMINING COEFFICIENT OF VISCOSITY OF

2.8. TERMINAL VELOCITY AND STOKES' FORMULA

Let us consider an infinite column of a highly viscous liquid like castor oil contained in a tall jar. If a steel ball is dropped into the liquid, it begins to move down with acceleration under gravitational pull. But its motion in the liquid is opposed by viscous forces in the liquid. These viscous forces increase as the velocity of the ball increases. Finally a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. At this stage, the resultant force on the ball is zero. Therefore the ball continues to move down with the same velocity thereafter. This uniform velocity is called the *terminal velocity.*

Stokes' Formula : The viscous force F experienced by a falling sphere must depend on (*i*) the terminal velocity v of the ball, (*ii*) the radius (r) of the ball and (*iii*) the coefficient of viscosity (η) of the liquid. We can write $F = k\, v^a\, r^b\, \eta^c$ where k is a dimensionless constant. The dimensions of these quantities are $F = MLT^{-2}$; $v = LT^{-1}$; $r = L$; $\eta = ML^{-1}T^{-1}$; (k is a number; it has no dimensions).

$$\therefore \quad MLT^{-2} = (LT^{-1})^a L^b (ML^{-1}T^{-1})^c$$

$$MLT^{-2} = M^c L^{a+b-c} T^{-a-c}$$

Equating the powers of M, L and T on either side,

$c = 1$; $a + b - c = 1$ and $-a - c = -2$.

Solving, $a = 1$; $b = 1$ and $c = 1$. $\therefore F = k\, v\, r\, \eta$.

Stokes experimentally found the value of k to be 6π.

$$\therefore \quad F = 6\pi v r \eta.$$

Expression for terminal velocity. Let ρ be the density of the ball and ρ' the density of the liquid. Then,

$$\text{the weight of the ball} = \frac{4}{3}\pi r^3 \rho g.$$

$$\left.\begin{array}{l}\text{The weight of the displaced liquid}\\ \text{or the upthrust on the ball}\end{array}\right\} = \frac{4}{3}\pi r^3 \rho' g$$

$$\left.\begin{array}{l}\text{The apparent weight}\\ \text{of the ball}\end{array}\right\} = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho' g = \frac{4}{3}\pi r^3 (\rho - \rho') g.$$

When the ball attains its terminal velocity v,

the apparent weight of the ball = viscous force F.

$$\therefore \quad 6\pi v r \eta = \frac{4}{3}\pi r^3 (\rho - \rho') g$$

or

$$v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho') g$$

Assumptions made by Stokes while deriving the formula:

(1) The medium through which the body falls is *infinite in extent.*

(2) The moving body is *perfectly rigid* and *smooth.*

(3) There is *no slip between the moving body and the medium.*

(4) There are no eddy currents or waves set up in the medium due to the motion of the body through it. In other words, the body is moving very slowly through it.

2.9. STOKES' METHOD FOR THE COEFFICIENT OF VISCOSITY OF A VISCOUS LIQUID

Stokes' method is suitable for highly viscous liquids like castor oil and glycerine. The experimental liquid is taken in a tall and wide jar (Fig. 2.7). Four or five marks *A, B, C, D*... are drawn on the outside of the jar at intervals of 5 cm. A steel ball is gently dropped centrally into the jar. The time taken by the ball to move through the distances *AB, BC, CD*, ... are noted. When the times for two consecutive transits are equal, the ball has reached terminal velocity. Now another ball is gently dropped into the jar. When the ball just reaches a mark below the terminal stage, the time (*t*) taken by the ball to move through a definite distance (*x*) is noted.

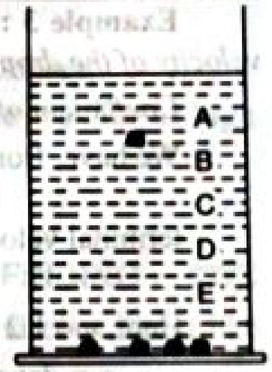


Fig. 2.7

∴ **Terminal velocity = $v = x/t$.**

The experiment is repeated for varying distances and the mean value of v is found.

The radius of the ball is measured accurately with a screw gauge. The density of the ball ρ and the density of the liquid ρ' are found by the principle of Archimedes. η is calculated using the formula

$$\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \rho')\,g$$

Example 1 : ***Assuming that when a spherical body moves in a viscous fluid under the action of a force F, the resultant force is given by $ma = F - 6\pi r\eta v$, calculate the terminal velocity of a rain drop of diameter 10^{-3} m. Density of air relative to water is 1.3×10^{-3}; $\eta = 1.81 \times 10^{-5}$ N sm^{-2}.***

When the body attains the terminal velocity, the acceleration of the body is zero. The body continues moving in the direction of the force with a constant velocity (v).

$$v = \frac{2}{9}\frac{r^2(\rho - \rho')g}{\eta};$$

Here $$r = \frac{10^{-3}}{2} = 5 \times 10^{-4}\ \text{m}$$

ρ = Density of water = 1000 kg m^{-3}.

ρ' = Density of air = Density of air relative to water × Density of water = $(1.3 \times 10^{-3}) \times 1000$ = 1.3 kg m^{-3}; $\eta = 1.81 \times 10^{-5}$ N sm^{-2} and $g = 9.8$ ms^{-2}.

∴ $$v = \frac{2}{9}\frac{(5 \times 10^{-4})^2\,(1000 - 1.3)\,9.8}{1.81 \times 10^{-5}} = 30\ \text{ms}^{-1}$$

DIFFUSION (2m)

Definition

The process by virtue of which the molecules of a solute move upward from the lower portion of greater concentration to upper portion of lower concentration is called diffusion.

Diffusion takes place in gases also. A tall glass jar is divided into two portions by a movable glass plate. The upper jar containing H2 is placed over a jar containing CO_2. When separation (glass plate) is removed in a short time, the molecules of the two gases mix with each other until a mixture of two gases are equal.

Experiments were carried out on diffusion by Graham and formulated several laws. These are known Garham's Law of diffusion. He arrived at the following conclusions:

1. The rate of diffusion increases with temperature.
2. The rate of diffusion depends upon the nature of the solute.
3. Diffusion alters the proportion of two salts in a mixture.

4. For the same substance of different concentration the rate of diffusion is directly proportional to the concentration.
5. Substances have been classified into
   1) crystalloids 2) Colloids

In Crystallojds diffusion takes place very fast. For eg. Salt solution, mineral acids etc. In colloids diffusion takes place very very slowly. For eg., gum, caramel, gelatine etc.

GRAHAM'S LAW FOR DIFFUSION OF GASES

Gases diffuse much more rapidly than liquids due to their high molecular speed. Gas diffusion is governed by Graham's law of diffusion.

The rates of diffusion of two gases are inversely proportional to the square roots of their densities. If r_1, r_2 & ρ_1, ρ_2 be the rates of diffusion and densities of two gases.

Then

$$\frac{r_1}{r_2} \propto \sqrt{\frac{1/\rho_1}{1/\rho_2}} = \frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

[$r_1 = 1/\rho_1$, $r_2 = 1/\rho_2$]

FICK'S LAW

Fick formulated a law of diffusion based on Fourier's law. It states that the amount of solute undergoing diffusion is,

1. directly proportional to area of cross section where diffusion takes place.
2. directly proportional to the time of diffusion
3. directly proportional to the concentration gradient of the solute in that direction

ie, $Q \propto A$ [Q = amount of solute]

$\propto t$

$\propto dc/dx$

$$Q \propto At(dc/dx)$$

$$Q = KAt\,(dc/dx)$$

Here,

$$\frac{dc}{dx} = \frac{c_1 - c_2}{x},$$

Where c_1, c_2 are the concentration of the two solution and x is the distance through which diffusion takes place.

$$Q = \frac{KAt\,(c_1 - c_2)}{X},$$

Where K is a diffusion constant also called as co-efficient of diffusion (or) diffusivity.

COEFFICIENT OF DIFFUSION – Diffusivity

The amount of solute passing across unit area of cross-section in unit time when concentration gradient is unity.

$$K = \frac{Qx}{At(C_1 - C_2)}$$

ANALOGY WITH HEAT CONDUCTION

Heat conduction in solids and diffusion in liquids are similar phenomena. Heat conduction,

$$Q = \frac{\lambda A (\theta_1 - \theta_2) t}{d}$$

Where θ is the quantity of heat flowing in 't' secs across the conductor, of area A, when heat pass from one layer to another layer of conductor. Then temperatures of two layers are θ_1, θ_2 and d is the distance between the two layers.

EFFUSION – TRANSPIRATION & TRANSFUSION

EFFUSION

When a gas escapes from a vessel into a vacuum through a small hole in the plate such that the diameter of the hole is greater than the length of the plate. Then the gas is said to effuse and this process is called effusion.

Graham showed that the rate of effusion is directly proportional to the square root of the difference on the two sides of the hole and inversely proportional to the square root of its der

$$\text{Velocity of effusion} \propto \sqrt{\frac{\text{Pressure difference}}{\text{Density}}}$$

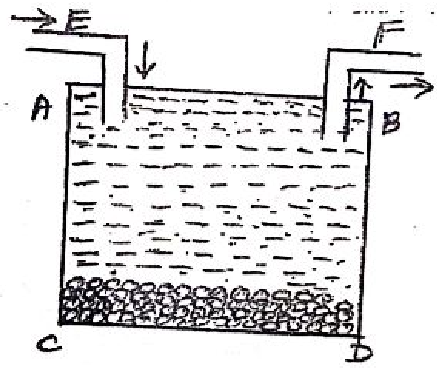
TRANSPIRATION

On the other hand, whenever a gas escapes from vessel into vacuum through the hole in a plate, such that the length of the plate is large compare to the diameter of the hole. Then the process of the gas escaping is called transpiration.

TRANSFUSION

When a gas escapes from the vessel into vacuum through a hole in the plate then diameter and length are comparable. This process is called transfusion.

EXPERIMENTAL DETERMINATION OF COEFFICIENT OF DIFFUSION (OR) DIFFUSIVITY.



Let ABCD is a vessel with sufficient quantity of the given salt at its bottom. Let CD be the concentration of the bottom layer. A dilute solution of the salt in water is poured gently into the vessel to a distance x. without disturbing the solution at the bottom a slow and steady flow of water is made to enter the vessel at A with the help of tube E. The water is coming out of the vessel through a tube F at B as shown. The flow of water should be so slow that it should not disturb the process of diffusion. After sometime, a steady state will be attained when the amount of the salt diffusing through the area AB at the top is constant. Then solution coming out of the vessel is collected in a vessel of known weight for known interval of time and then amount of salt present in it is calculated.

Then concentration C_2 at the top most layer AB is calculated. By a preliminary experiment, the concentration C_1 at the bottom most layer CD is also known. If Q be the amount of the given solute and area of the topmost layer is 'A', then by Fick's Law,

$$Q = KAT\ (C_1 - C_2)\ /\ x$$

Here, Q, A, t and $(C_1 - C_2) / x$ are known

Then,

$$K = \frac{Q}{At\,(C_1 - C_2)\ /\ x}$$ is calculated.

APPLICATION OF DIFFUSION

1. Diffusion is a very important process in making integrated circuits (IC) used in the electronic industry to construct tiny devices. For example diodes, transistors etc. It is necessary to create controlled n & p type regions in the silicon. This is usually done using a process known as solid state diffusion.

2. Diffusion principle is used in cloud chamber which can be used for detecting or photographing ionizing particles and radiation.

3. Diffusion process is used to manufacture the chemical substances.