21.06.19

CLASSICAL ALGEBRA

SYLLABUS

UNIT - I

Theory of Equations ①Irrational roots of equation with rational coefficients -②Relations connecting the roots and coefficients (Roots in A.P, G.P, H.P, Product of roots, Sum of the roots in terms coefficients only) -③Sum of the power of the roots of an equation -④Newton's theorem on the power of the root.

UNIT - II

Roots multiplied by a given number - Standard reciprocal equation (with coefficients have like signs, unlike signs, odd sign or degree, even degree - diminishing and increasing the roots of an equation by a given number - Descartes rule of signs - Roll's theorem - Multiple rule ~~Newton's and~~ Horner's Method of Approximations (Upto two decimals only).

Compulsory question

UNIT - III

Binomial, Exponential and Logarithmic's theorem - Applications of Binomial theorem on summation problems only - Applications of

Exponential theorem on summation problems only - Applications of Logarithmic Theorem on summation problems only -

UNIT - IV

Convergence of Sequences and Series

Definition and Elimentary Results - Comparison test - Ratio test - De-Alembert's and Cauchy's Test.

UNIT - V

Absolute Convergence Series of Positive terms - Cauchy's condensation Test - Raabe's Test - Grauss Test - De-Morgan and Bertrand Test.

TEXT BOOK

1. Algebra by T.K.M. Pillai and others

REFERENCE BOOK

2. Algebra by T. Natarajan and others.
3. Algebra by Hall and Knight.

24.06.2019

UNIT - I

(1)

Theory of Equations :-

Definition :-

The complex number is of the form $a+ib$, where a is a real part and b is a imaginary part.

Property 1 :-

In an equation, with real coefficient complex roots occurs in pairs.

Property 2 :-

In an equation, with rational coefficient irrational roots occur in pairs.

Problems :-

1. Solve the equation $x^4+2x^3-5x^2+6x+2=0$ given that $1+\sqrt{-1}$ is a root of it.

Solution :-

The Given equation is

$$x^4+2x^3-5x^2+6x+2=0 \quad \text{——— (1)}$$

The Highest power of equation (1) is 4

$\therefore$ It has four roots

Given that : $1+i$ is a root of it

therefore $1-i$ is also a root (imaginary roots occurs in pairs)

the factors are

$$[x-(1+i)] \ \& \ [x-(1-i)]$$

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Multiplying the factors

$$[x-(1+i)]\,[x-(1-i)]$$

$x^2 - 1x + ix$

$\Rightarrow (x-1-i)(x-1+i)$

$\Rightarrow ((x-1)-i)\,((x-1)+i)$

$\Rightarrow (x-1)^2 - (i)^2$

$\Rightarrow x^2 - 2x + 1 + 1$

$\Rightarrow x^2 - 2x + 2$ ———— ②

Divide ① by ②

$$
\begin{array}{r|l}
 & x^2 + 4x + 1 \\
\hline
x^2-2x+2 & x^4 + 2x^3 - 5x^2 + 6x + 2 \\
(-) & x^4 \overset{(-)}{-} 2x^3 \overset{(-)}{+} 2x^2 \\
\hline
 & 4x^3 - 7x^2 + 6x \\
 & 4x^3 \overset{(+)}{-} 8x^2 \overset{(-)}{+} 8x \\
\hline
 & x^2 - 2x + 2 \\
 & x^2 - 2x + 2 \\
\hline
 & 0
\end{array}
$$

The Remainder $x^2 + 4x + 1 = 0$ ———— ③

Solving ③ $a = 1 \quad b = 4 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$x = \frac{-4 \pm \sqrt{12}}{2}$$

$4 <$

$\overline{1}$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= -2 \pm \sqrt{3}$$

$\therefore$ The roots are $1+i, 1-i, -2+\sqrt{3}$ and $-2-\sqrt{3}$

---

2. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ Given that $1-\sqrt{5}$ is a root of it

Solution.

The given equation is

$$x^4 - 5x^3 + 4x^2 + 8x - 8 = 0 \qquad \text{—— (1)}$$

The Highest power of equation (1) is 4

$\therefore$ It has four roots.

Given that :- $1-\sqrt{5}$ is a root

$\therefore$ $1+\sqrt{5}$ is also a root

(irrational roots occur in pairs)

The factor are

$[x-(1-\sqrt{5})]$ & $[x-(1+\sqrt{5})]$

Multiplying the factors

$$[x-(1-\sqrt{5})]\ [x-(1+\sqrt{5})]$$

$$\Rightarrow (x-1+\sqrt{5})(x-1-\sqrt{5})$$

$$\Rightarrow ((x-1)+\sqrt{5})((x-1)-\sqrt{5})$$

$$\Rightarrow (x-1)^2 - (\sqrt{5})^2$$

$$\Rightarrow x^2 - 2x + 1 - 5$$

$$\Rightarrow x^2 - 2x - 4 \qquad \text{—— (2)}$$

Divide ① by ②

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 2x - 4 & x^4 - 5x^3 + 4x^2 + 8x - 8 = 0 \\
 & \underline{{}_{(-)}x^4 \underset{(+)}{-} 2x^3 \underset{(+)}{-} 4x^2} \\
 & \qquad -3x^3 + 8x^2 + 8x \\
 & \qquad \underline{\underset{(+)}{-}3x^3 \underset{(-)}{+} 6x^2 \underset{(-)}{+} 12x} \\
 & \qquad\qquad 2x^2 - 4x - 8 \\
 & \qquad\qquad \underline{{}_{(-)}2x^2 \underset{(+)}{-} 4x \underset{(+)}{-} 8} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

The remaining equation is

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x+3) = 0$$

$$x - 1 = 0 \qquad \cancel{x+3=0} \qquad x - 2 = 0$$

$$x = 1 \qquad \cancel{x = +3} \qquad x = +2$$

$$\boxed{x = 1, +2}$$

Therefore the roots are $1, +2, 1-\sqrt{5}$ and $1+\sqrt{5}$.

3. Solve the equation.

$$x^4 - 4x^2 + 8x + 35 = 0$$

Given that $2+i\sqrt{3}$ is a root of it.

4. Solve the equation.

$$3x^3 - 4x^2 + x + 88 = 0$$

Given that $2-i\sqrt{7}$ is a root of it.

Given equation is

$$3x^3 - 4x^2 + x + 88 = 0 \quad \text{——— (1)}$$

The highest power of equation (1) is 3

$\therefore$ It has three roots

Given :- $2-i\sqrt{7}$ is a root.

$\therefore$ $2+i\sqrt{7}$ is also a root

(imaginary roots occur in pairs).

The factors are

$$(x - (2+i\sqrt{7}))(x-(2-i\sqrt{7}))$$

$$\Rightarrow (x-2)^2 - (i\sqrt{7})^2$$

$$\Rightarrow x^2 - 4x + 4 - i^2(7).$$

$$\Rightarrow x^2 - 4x + 11 \quad \text{——— (2)}$$

(1)/(2) :

$$
\begin{array}{r|l}
 & \quad 3x + 8 \\
\hline
x^2-4x+11 & 3x^3 - 4x^2 + x + 88 \\
 & \underset{(-)}{3x^3} \underset{(+)}{-12x^2} \underset{(-)}{+33x} \\
\hline
 & \qquad 8x^2 - 32x + 88 \\
 & \qquad 8x^2 - 32x + 88 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

The remaining equation is

$$3x + 8 = 0$$

$$3x = -8$$

$$x = -8/3$$

The roots are $2 - i\sqrt{7}$, $2 + i\sqrt{7}$ and $-\frac{8}{3}$

5. Solve the equation

$$3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$$

Given that $\sqrt{2} + \sqrt{5}$ is a root of it

Solution

Given that :-

$$3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0 \quad \text{—(1)}$$

Given that $\sqrt{2} + \sqrt{5}$ is a root of it.

$\therefore \sqrt{2} - \sqrt{5}$, $-\sqrt{2} + \sqrt{5}$, $-\sqrt{2} - \sqrt{5}$ is also a root

~~Multiplying~~ the factor are

$$x - (\sqrt{2}+\sqrt{5}),\ x - (\sqrt{2} - \sqrt{5}),\ x - (-\sqrt{2} + \sqrt{5}),$$
$$(x - (-\sqrt{2} - \sqrt{5})$$

Multiplying the factor.

$$(x - (\sqrt{2}+\sqrt{5}))\ (x - (\sqrt{2}-\sqrt{5}))(x - (-\sqrt{2}+\sqrt{5}))(x - (-\sqrt{2}-\sqrt{5})$$

$$\Rightarrow ((x-\sqrt{2}) - \sqrt{5})\ ((x-\sqrt{2}) + \sqrt{5})((x+\sqrt{2}) - \sqrt{5})((x+\sqrt{2}) + \sqrt{5})$$

$$\Rightarrow (x-\sqrt{2})^2 - (\sqrt{5})^2\ (x+\sqrt{2})^2 - (\sqrt{5})^2$$

$$\Rightarrow (x^2 - 2\sqrt{2}x + 2 - 5)\ \ (x^2 + 2\sqrt{2}x + 2 - 5)$$

$$\Rightarrow (x^2 - 2\sqrt{2}x - 3)\ (x^2 + 2\sqrt{2}x - 3)$$

$\Rightarrow$

$$\Rightarrow (x^2 - 2\sqrt{2}x - 3)(x^2 + 2\sqrt{2}x - 3)$$

$$\Rightarrow (x^4 + \cancel{2\sqrt{2}x^3} - 3x^2 - \cancel{2\sqrt{2}x^3} - 8x^2 + \cancel{6\sqrt{2}x}$$

$$-3x^2 - \cancel{6\sqrt{2}x} + 9$$

$$\Rightarrow x^4 - 14x^2 + 9 \quad \text{———} \quad (2)$$

Divide (1) by (2)

$$\begin{array}{r|l} & \quad 3x - 4 \\ \hline x^4 + 0x^3 - 14x^2 + 0x + 9 & 3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 \\ & 3x^5 + 0x^4 - 42x^3 + 0x^2 + 27x \\ & (-) \quad (-) \quad (+) \quad (-) \quad (-) \\ \hline & -4x^4 \qquad + 56x^2 - 36 \\ & -4x^4 \qquad + 56x^2 - 36 \\ \hline & \qquad\qquad 0 \end{array}$$

the remaining root are.

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

the roots are $\sqrt{2} + \sqrt{5}$, $\sqrt{2} - \sqrt{5}$, $-\sqrt{2} + \sqrt{5}$, $-\sqrt{2} - \sqrt{5}$ and $4/3$.

Monday

3 Solve the equation

$$x^4 - 4x^2 + 8x + 35 = 0$$

Given that $2 + i\sqrt{3}$ is a root of it.

Solution :-

The given equation is

$$x^4 - 4x^2 + 8x + 35 = 0 \quad \text{———} \quad (1)$$

The Highest power of equation (1) is 4.

$\therefore$ It has four roots.

Given that : $2 + i\sqrt{3}$ is a root

$\therefore$ $2 - i\sqrt{3}$ is also a root

(Imaginary roots occur in pairs)

The factors are $[x - (2 + i\sqrt{3})]$ & $[x - (2 - i\sqrt{3})]$

X $\Rightarrow (x - (2 + i\sqrt{3}))(x - (2 - i\sqrt{3}))$ [Multiplying the factors]

$\Rightarrow ((x-2) + i\sqrt{3})((x-2) - i\sqrt{3})$

$\Rightarrow (x-2)^2 - (i\sqrt{3})^2$

$\Rightarrow x^2 - 4x + 4 - (-1)(3)$

$\Rightarrow x^2 - 4x + 4 + 3$

$\Rightarrow x^2 - 4x + 7$

The remaining equation is $x^2 - 4x + 7 = 0$ — (2)

Divide equation (1) by (2)

$$
\begin{array}{r|l}
 & x^2+4x+5 \\
\hline
x^2-4x+7 & x^4+0x^3-4x^2+8x+35 \\
 & x^4-4x^3+7x^2 \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 4x^3-11x^2+8x \\
 & 4x^3-16x^2+28x \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 5x^2-20x+35 \\
 & 5x^2-20x+35 \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 0
\end{array}
$$

$$
\begin{array}{r|l}
 & x^2+4x+5 \\
\hline
x^2-4x+7 & x^4+0x^3-4x^2+8x+35 \\
 & x^4-4x^3+7x^2 \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 4x^3-11x^2+8x \\
 & 4x^3-16x^2+28x \\
 & (-)\quad(+)\quad(-) \\
\hline
 & 5x^2-20x+35 \\
 & 5x^2-20x+35 \\
\hline
 & 0
\end{array}
$$

The remaining equation is

$a=1 \quad b=4 \quad c=5$

$x = \dfrac{-4 \pm \sqrt{16-4(1)(5)}}{2}$

$= -4 \pm \sqrt{(-1)4}/2$

$= \dfrac{-4 \pm i2}{2}$

$= -2 \pm i$

$\cancel{x^2+4x+5=0}$

$\cancel{(x+1)(x+4)=0}$

$\cancel{x+1=0} \quad | \quad \cancel{x+4=0}$

$\cancel{x=-1} \quad \cancel{x=-4}$

$\cancel{x=-1,-4}$

$4 < \begin{array}{l} 1 \\ 4 \end{array}$

$\dfrac{4}{4}$

$\therefore$ The roots are $2+i\sqrt{3},\ 2-i\sqrt{3},\ -2 \pm i$.

[illegible]

Solve the equation

$$2x^6 - 3x^5 + 5x^4 + 6x^3 - 27x + 81 = 0$$

Given that $\sqrt{2} - \sqrt{-1}$ is a root of it

Solution

The factors are $\sqrt{2} - i$, $\sqrt{2} + i$, $-\sqrt{2} - i$ and $-\sqrt{2} + i$

Multiplying the factor.

$$(x - (\sqrt{2} - i))(x - (\sqrt{2} + i))\cdot(x - (-\sqrt{2} - i))(x - (-\sqrt{2} + i))$$

$$((x - \sqrt{2}) - i)((x - \sqrt{2}) + i)((x + \sqrt{2}) + i)((x + \sqrt{2}) - i)$$

$$\Rightarrow ((x - \sqrt{2})^2 - i^2)((x + \sqrt{2})^2 - i^2)$$

$$\Rightarrow (x^2 - 2\sqrt{2}x + 2 + 1)(x^2 + 2\sqrt{2}x + 2 + 1)$$

$$\Rightarrow (x^2 - 2\sqrt{2}x + 3)(x^2 + 2\sqrt{2}x + 3)$$

$$\Rightarrow x^4 + \cancel{2\sqrt{2}x^3} + 3x^2 \cancel{- 2\sqrt{2}x^3} - 8x^2 \cancel{- 6\sqrt{2}x}$$
$$+ 3x^2 \cancel{+ 6\sqrt{2}x} + 9$$

$$\Rightarrow x^4 + 6x^2 - 8x^2 + 9$$

$$\Rightarrow x^4 - 2x^2 + 9 \quad \text{———} \quad (2)$$

Divide by (1) by (2)

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 9 \\
\hline
x^4 + 0x^3 - 2x^2 + 0x + 9 & 2x^6 - 3x^5 + 5x^4 + 6x^3 + 0x^2 - 27x + 81 \\
 & 2x^6 + 0x^5 - 4x^4 + 0x^3 + 18x^2 \\
 & (-) \quad (-) \quad (+) \quad (-) \quad (-) \\
\hline
 & -3x^5 + 9x^4 + 6x^3 - 18x^2 - 27x \\
 & -3x^5 + 0x^4 + 6x^3 + 0x^2 - 27x \\
 & (+) \quad (-) \quad (-) \quad (+) \\
\hline
 & 9x^4 - 18x^2 + 81 \\
 & 9x^2 - 18x^2 + 81 \\
 & (-) \quad (+) \quad (-) \\
\hline
 & 0
\end{array}
$$

The remaining equation is

$$2x^2 - 3x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(9)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 - 72}}{4}$$

72
9
63

$$= \frac{3 \pm \sqrt{9 + (-1)(72)}}{4}$$

$$\cancel{= \frac{3\sqrt{9} \pm i\sqrt{72}}{4}}$$

$$= \frac{3 \pm \sqrt{-63}}{4}$$

$$= \frac{3 \pm i\sqrt{3 \times 3 \times 7}}{4}$$

$$= \frac{3}{4} \pm \frac{i\sqrt{63}}{4}$$

$$= \frac{3}{4} \pm \frac{i3\sqrt{7}}{4}$$

$$= \frac{3}{4} \pm \frac{i3\sqrt{7}}{4}$$

∴ The roots are

$\sqrt{2} - i, \sqrt{2} + i, -\sqrt{2} - i, -\sqrt{2} + i, \frac{3}{4} + \frac{i3\sqrt{7}}{4}$ and $\frac{3}{4} - \frac{i3\sqrt{7}}{4}$

② Relation between the roots and coefficients

$ax^n + bx^{n-1} + cx^{n-2} + \dots\dots = 0$ is an equation if it has the roots $\alpha, \beta, \gamma, \delta$

(i) $\Sigma\alpha = \alpha + \beta + \gamma + \dots\dots$

$$\boxed{S_1 = \Sigma\alpha = -\frac{2^{nd}\ \text{coefficient}}{1^{st}\ \text{coefficient}}}$$

$$= -\frac{b}{a}$$

(ii) $\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\delta + \alpha\delta + \alpha\gamma + \dots$

$$\boxed{S_2 = \Sigma\alpha\beta = \frac{3^{rd}\ \text{coefficient}}{1^{st}\ \text{coefficient}}}$$

$$= \frac{c}{a}$$

(iii) $\Sigma\alpha\beta\gamma = \alpha\beta\gamma + \alpha\beta\delta + \alpha\beta\gamma + \dots\dots$

$$\boxed{= -\frac{4^{th}\ \text{coefficient}}{1^{st}\ \text{coefficient}}}$$

$$S_3 = \Sigma\alpha\beta\gamma = -\frac{d}{a}$$

(iv) $\Sigma\alpha\beta\gamma\delta = \alpha\beta\gamma\delta + \dots\dots$

$$\boxed{S_4 = \Sigma\alpha\beta\gamma\delta = +\frac{5^{th}\ \text{coefficient}}{1^{st}\ \text{coefficient}}}$$

$$= e/a$$

In general,

If $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$ is an equation then

$$S_n = (-1)^n \frac{a_n}{a_0} \text{ or}$$

$n=1$ $S_1 = (-1)^1 \frac{b}{a} = -b/a$

1m V.I

If $\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n$ are the roots of the equation

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n = 0$$

Then $\Sigma \alpha_1 \alpha_2 \ldots \alpha_n$ is equal to or

$$\boxed{S_n = (-1)^n \frac{a_n}{a_0}.}$$ or Put $n=1$ $S_1 = (-1)^1 \frac{b}{a}$

$$x_{n+1} = x_n - \frac{p(x_n)}{p'(x_n)}$$

(2)

Types of roots	For Cubic Equation	For 4^{th} degree equation
A.P $S_2 = 3a^2 - d^2 = \frac{3^{rd}}{1^{st}}$	Take the roots as $\boxed{(a-d), a, (a+d)}$ use the formula. $S_1 = a-d+a+a+d$ $= \frac{-2^{nd} \text{ coefficient}}{1^{st} \text{ coefficient}}$	Take the roots as $\boxed{(a-3d)(a-d)(a+d)(a+3d)}$ Use the formula $S_2 = 6a^2 - 10d^2$ $= \frac{3^{rd} \text{ coefficient}}{1^{st} \text{ coefficient}}$
G.P	Take the roots as $\boxed{\frac{a}{r}, a, ar}$ Use the formula $\Sigma \alpha\beta\gamma$ $S_3 = \frac{a}{r} a \, ar$ $= \frac{-4^{th} \text{ coeff}}{1^{st} \text{ coeff}}$	Take the roots as $\boxed{\alpha, \beta, \gamma, \delta}$ and use the formula $\alpha\delta = \beta\gamma$ $= \frac{+5^{th} \text{ coeffi}}{1^{st} \text{ coeffi}}$
H.P	Take the roots as $\boxed{\alpha, \beta, \gamma}$ Use the formula. $\boxed{\frac{2}{\beta} = \frac{1}{\alpha} + \frac{1}{\gamma}}$ $\frac{2}{\beta} = \frac{1}{\alpha} + \frac{1}{\gamma}$	-

The roots are $-\frac{1}{2}, 2, \frac{9}{2}$

2 $x^3 - 6x^2 + 11x - 6 = 0$ whose roots in AP

Solution :-

$$x^3 - 6x^2 + 11x - 6 = 0 \quad \text{———} \quad (1)$$

The three roots are $a-d, a, a+d$.

Step 1 :

$S_1 : \Sigma\alpha : a-d+a+a+d = \frac{-(-6)}{1}$

$3a = 6$

$\boxed{a = 2}$

Step 2 :-

Removing the factor $x-2$ from (1) we get

$$\begin{array}{c|cccc} 2 & 1 & -6 & +11 & -6 \\ & 0 & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\boxed{x = 1}$ and $\boxed{x = 3}$

$3 < \begin{matrix} -1 \\ -3 \end{matrix}$, -4

The roots are $1, 2, 3$

Type - II four roots are in A.P

1\. Solve $x^4 + 2x^3 - 21x^2 - 22x + 40$.

Solution

Let The four roots are $(a-3d), (a-d), a+d$

$(a+3d)$

Step 1 :- $S_1 = \Sigma\alpha = a-3d+a-d+a+d+a+3d = \frac{-2^{nd}\text{ co}}{1^{st}\text{ coef}}$

$$4a = -2$$

$$\boxed{a = \frac{-1}{2}}$$

Step 2 :-

$$\Sigma\alpha\beta = (a-3d)(a-d) + (a-3d)(a+d) + (a-3d)(a+3d) + (a-d)(a+3d) + (a-d)(a+d) + (a+d)(a+3d) = \frac{3^{rd}\text{ coef}}{1^{st}\text{ coef}}$$

$$6a^2 - 10d^2 = -21 \quad \text{---} \ (*)$$

Put $a = -\frac{1}{2}$ in $(*)$

$$\cancel{6}^{3}\left(\frac{1}{\cancel{4}_2}\right) - 10d^2 = -21$$

$$\frac{3}{2} - 10d^2 = -21$$

$$\frac{3 - 20d^2}{2} = -21$$

$$-20d^2 = -42 - 3$$

$$d^2 = \frac{\cancel{-45}\ 9}{\cancel{-20}\ 4}$$

$$d^2 = \frac{9}{4} \qquad \boxed{d = \pm\frac{3}{2}}$$

The roots are $a-3d, a-d, a+d, a+3d$

$a = -1/2 \quad d = 3/2$

$-\frac{1}{2} - 3\left(\frac{3}{2}\right), \frac{-1}{2} - \frac{3}{2}, \frac{-1}{2} + \frac{3}{2}, \frac{-1}{2} + 3\left(\frac{3}{2}\right)$

$= \left(\frac{-1}{2} - \frac{9}{2}\right), \left(\frac{-1-3}{2}\right), \frac{-1+3}{2}, \left(\frac{-1+9}{2}\right)$

$= -\frac{10}{2}, -\frac{4}{2}, \frac{2}{2}, \frac{8}{2}. \quad = -5, -2, 1, 4$

2. $16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0$ the roots are in A.P.

3. $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ the roots are in A.P.

2. $16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0.$

Solution :-

Let the 4 roots are $a-3d, a-d, a+d, a+3d$.

Step 1 :

$S_1 = \Sigma\alpha = a-3d + a-d + a+d + a+3d = -\frac{2^{nd} \text{ coef}}{1^{st} \text{ coef}}$

$4a = \frac{-(-64)}{16}$

$a = \frac{64}{16 \times 4}$

$a = \frac{64}{64}$

$\boxed{a = 1}$

Step 2 :-

$S_2 = \Sigma \alpha\beta = (a-3d)(a-d) + (a-3d)(a+d)$
$+ (a-3d)(a+3d) + (a-d)(a+3d)$
$+ (a-d)(a+d) + (a+d)(a+3d) = \frac{3^{rd} \text{ coeff}}{1^{st} \text{ coeff}}$

$$6a^2 - 10d^2 = \frac{56}{16} \quad \text{—} \circledast$$

Put $a=1$ in $\circledast$

$$6(1) - 10d^2 = \frac{56}{16}$$

$$6 - 10d^2 = \frac{56}{16}$$

$$-10d^2 = \frac{56}{16} - 6$$

$$= \frac{56-96}{16}$$

$$\begin{array}{r} \overset{3}{16} \times 6 \\ \hline 96 \end{array} \qquad \begin{array}{r} -96 \\ 56 \\ \hline -40 \end{array}$$

$$\cancel{-}10d^2 = \cancel{-}\frac{40}{16}$$

$$d^2 = \frac{4\cancel{0}}{16\cancel{0}}$$

$$d^2 = \frac{2^2}{4^2}.$$

$$d = \frac{\cancel{2}}{\cancel{4}} \quad \boxed{d = \pm 1/2}.$$

The roots are $a-3d, a-d, a+d, a+3d$.

$a = 1 \quad d = 1/2$

$= 1 - 3/2, \; 1 - 1/2, \; 1 + 1/2, \; 1 + 3/2$

$= \frac{-1}{2}, \; 1/2, \; 3/2, \; 5/2$ //

3. $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ A.P

Solution.

Let the 4 roots are $a-3d, a-d, a+d, a+3d$.

Step 1:

$S_1 = \Sigma\alpha = a-3d+a-d+a+d+a+3d = \dfrac{-2^{nd}\text{ coef}}{1^{st}\text{ coef}}$

$$4a = -(-8)$$

$$4a = 8$$

$$\boxed{a = 2}$$

Step 2:-

$S_2 = \Sigma\alpha\beta = (a-3d)(a-d) + (a-3d)(a+d)$
$+ (a-3d)(a+3d) + (a-d)(a+3d)$
$+ (a-d)(a+d) + (a+d)(a+3d) = \dfrac{3^{rd}\text{ coef}}{1^{st}\text{ coef}}$

$$6a^2 - 10d^2 = 14 \quad \text{—} \circledast$$

Put $a=2$ in $\circledast$

$$6(4) - 10d^2 = 14$$

$$24 - 10d^2 = 14$$

$$-10d^2 = 14 - 24$$

$$d^2 = \frac{-10}{-10}$$

$$d^2 = 1 \qquad d = \pm 1$$

the roots are

$a=2 \quad d=1$.

$= 2-3, 2-1, 2+1, 2+3$

$= -1, 1, 3, 5$ //

Type - iii The 3 roots are in G.P

1. Solve the equation.

$x^3 - 19x^2 + 114x - 216 = 0$ whose roots are in G.P

Solution :

Let the 3 roots are $\frac{a}{r}, a, ar$

Step 1.

$S_3 = \Sigma\alpha\beta\gamma$: $\frac{a}{r} \times a \times ar = \frac{-4^{th} \text{ coef}}{1^{st} \text{ coef}}$

$$a^3 = -(-216)$$

$$a^3 = 216$$

$$\boxed{a = 6}$$

Step 2 :-

Removing the factor $x-6$ by (1)

6	1	-19	114	-216
	0	6	-78	216
	1	-13	36	0

-13×6
-78

$$x^2 - 13x + 36 = 0$$

$$(x-9)(x-4) = 0$$

$$x - 9 = 0 \quad x - 4 = 0$$

$$x = 9, 4.$$

$36 \left< \begin{matrix} -9 \\ -4 \end{matrix} \right.$
-13

The roots are 4, 6, 9

2 $8x^3 - 14x^2 + 7x - 1 = 0$ whose roots are in G.P

Solution :-

Let the three roots are $\frac{a}{r}, a, ar$

Step 1

$S_3 : \Sigma \alpha\beta\gamma : \frac{a}{r} \times a \times ar = \frac{-4^{th} \text{ coef}}{1^{st} \text{ coef}}$

$$a^3 = \frac{-(-1)}{8}$$

$$a^3 = \frac{1}{8}$$

$$\boxed{a = \frac{1}{2}}$$

Step 2 :-

Removing the factor.

$$\begin{array}{c|cccc} \frac{1}{2} & 8 & -14 & 7 & -1 \\ & 0 & 4 & -5 & 1 \\ \hline & 8 & -10 & 2 & \boxed{0} \end{array}$$

$$8x^2 - 10x + 2 = 0$$

$\div 2$ $$4x^2 - 5x + 1 = 0$$

$$4 \begin{array}{l} -1/4 \\ -4/4 \end{array} \; \begin{array}{c} 1 \\ \hline -5 \end{array}$$

$$(4x-1)(x-1) = 0$$

$$4x - 1 = 0 \quad x - 1 = 0.$$

$$x = 1/4 \quad x = 1$$

The roots are $1, 1/2, 1/4$

1 H.W: $27x^3 + 42x^2 - 28x - 8 = 0$ the roots are in G.P

Solution :-

Let the three roots are $\frac{a}{r}, a, ar$

$S_3 : \Sigma\alpha\beta\gamma : \frac{a}{r} \times a \times ar = \frac{-4^{th} \text{ coeffi}}{1^{st} \text{ coeffi}}$

$$a^3 = -(-8)/27$$

$$a^3 = 8/27$$

$$\boxed{a = 2/3}$$

Step 2 :-

Removing the factor.

$$\begin{array}{c|cccc} & 27 & 42 & -28 & -8 \\ 2/3 & 0 & 18 & 40 & 8 \\ \hline & 27 & 60 & +12 & \boxed{0} \end{array}$$

$40 - 28 = 12$

$$27x^2 + 60x + 12 = 0$$

$\div 3$ $$9x^2 + 20x + 4 = 0$$

$$(9x+2)(x+2) = 0$$

$36 < 18/9,\ 2/9$; 20

$9x + 2 = 0 \qquad x + 2 = 0$

$x = -\frac{2}{9} \qquad x = -2$

the roots are $-2, -\frac{2}{9}, \frac{2}{3}$

2. $x^3 - 7x^2 + 14x - 8 = 0$ whose roots are in G.P

Solution :-

Let the three roots are $a/r, a, ar$

$S_3 : \Sigma\alpha\beta\gamma : \frac{a}{r} \times a \times ar = \frac{-4^{th} \text{ coefficient}}{1^{st} \text{ coefficient}}$

$$a^3 = -(-8)/1$$

$$a^3 = 8$$

$$\boxed{a = 2}$$

Step 2 :-

Removing the factor

$$\begin{array}{c|cccc} & 1 & -7 & 14 & -8 \\ 2 & & & & \\ & 0 & 2 & -10 & 8 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array}$$

$$x^2 - 5x + 4 = 0$$

$$4 \begin{array}{l} -1 \\ -4 \end{array}$$

$$-5$$

$$(x-1)(x-4) = 0$$

$$x - 1 = 0 \qquad x - 4 = 0$$

$$\boxed{x = 1} \qquad \boxed{x = 4}$$

The roots are 1, 2, 4

3) $3x^3 - 26x^2 + 52x - 24 = 0$ whose roots are in G.P

Solution :

The three roots are $a/r, a, ar$

$S_3 : \Sigma\alpha\beta\gamma \quad \frac{a}{r} \times a \times ar = \frac{-4^{th}\ \text{coeffi}}{1^{st}\ \text{coeffi}}$

$$a^3 = \frac{-(-24)}{3}$$

$$a^3 = 8$$

$$\boxed{a = 2}$$

Step 2 :

Removing the factor

$$\begin{array}{c|cccc} 2 & 3 & -26 & 52 & -24 \\ & 0 & 6 & -40 & 24 \\ \hline & 3 & -20 & 12 & \boxed{0} \end{array}$$

$$3x^2 - 20x + 12 = 0.$$

$$36 < \begin{matrix} -18/\cancel{3}\ 6 \\ -2/3 \end{matrix}$$

$$-20$$

$$(x-6)(3x-2) = 0$$

$$x - 6 = 0 \qquad 3x - 2 = 0$$

$$\boxed{x = 6} \qquad \boxed{x = 2/3}$$

The roots are $6, 2, 2/3$

28.6.19 **Type - III**

The 4 roots are in G.P

Solve the equation

$x^4 - 30x^3 + 280x^2 - 960x + 1024 = 0$. The roots are in G.P.

Solution :-

The four roots are $\alpha, \beta, \gamma, \delta$

$$\boxed{\alpha\delta = \beta\gamma} \quad \text{— (1)}$$

$$x^4 - 30x^3 + 280x^2 - 960x + 1024 = 0 \quad \text{— (2)}$$

$$S_1 = \Sigma\alpha : \alpha + \beta + \gamma + \delta = -\frac{2^{nd}\text{ coef}}{1^{st}\text{ coef}}$$

$$(\alpha + \delta) + (\beta + \gamma) = -(-30) = 30 \quad \text{— (1)}$$

$$S_2 = \Sigma\alpha\beta : \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \delta\gamma = +\frac{3^{rd}\text{ coef}}{1^{st}\text{ coef}}$$

$$\alpha(\beta+\gamma) + \delta(\beta+\gamma) + \alpha\delta + \beta\gamma = 280$$

$$(\alpha+\delta)(\beta+\gamma) + \alpha\delta + \beta\gamma = 280$$

Sub (1) $\alpha\delta = \beta\gamma$

$$(\alpha+\delta)(\beta+\gamma) + 2\alpha\delta = 280 \quad \text{— (3)}$$

$$S_3 = \Sigma\alpha\beta\gamma = \alpha\beta\gamma + \beta\gamma\delta + \alpha\beta\delta + \alpha\gamma\delta = -\frac{4^{th}\text{ coef}}{1^{st}\text{ coef}}$$

by (1)

$$\beta\gamma(\alpha+\delta) + \alpha\delta(\beta+\gamma) =$$

$S_4 : \alpha\beta\gamma\delta = \frac{5^{th}}{1^{st}}$

$(\alpha\beta)^2 = 1024. \quad (\because \alpha\delta = \beta\gamma)$

$\therefore \quad \alpha\beta = 32.$ ——— (4)

Subs (4) in (3)

$(\alpha + \cancel{\beta}\delta)(\beta + \gamma) = 280 - 64$

$= 216$

$\begin{array}{r} 280 \\ 64 \\ \hline 216 \end{array}$

Consider.

$\boxed{(a-b)^2 = (a+b)^2 - 4ab}$

$(\alpha+\delta) - (\beta+\gamma) = \sqrt{(\alpha+\delta) + (\beta+\gamma)^2 - 4(\alpha+\delta)(\beta+\gamma)}$

$= \sqrt{(30)^2 - 4 \times 216}$

$= \sqrt{900 - 864}$

$= \sqrt{36}$

$\begin{array}{r} 216 \\ 864 \\ 900 \\ 864 \\ \hline 36 \end{array}$

$\boxed{(\alpha+\delta) - (\beta+\gamma) = 6}$ ——— (5)

(5) + (1) $\quad (\alpha + \beta) - (\beta + \gamma) = 6$ — (5)

$(\alpha + \delta) + (\beta + \gamma) = 30$ ——— (1)

$2(\alpha + \delta) = 36$

$\boxed{\alpha + \delta = 18}$

(5) − (1)

$-2(\beta + \gamma) = -24$

$\boxed{\beta + \gamma = 12}$

We know that

$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$ — (6)

$\alpha \cdot \delta = 32 \qquad \alpha + \delta = 18$

Take (6) becomes,

$x^2 - 18x + 32 = 0$

$(x-16)(x-2) = 0$

$32 \begin{smallmatrix} -16 \\ -2 \end{smallmatrix}$, -18

$\boxed{x = 16, 2}$.

Take $\beta\gamma = 32 \quad \beta + \gamma = 12$.

(6) becomes,

$x^2 - 12x + 32 = 0$

$(x-8)(x-4) = 0$

$32 \begin{smallmatrix} -8 \\ -4 \end{smallmatrix}$, 12

$x = 8 \qquad x = 4$

$x = 8, 4$.

∴ The roots are $2, 4, 8, 16$

2. Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$ The roots are in G.P.

Solution :

The four roots are $\alpha, \beta, \gamma, \delta$

$\alpha\delta = \beta\gamma$

$$x^4 + 15x^3 + 70x^2 + 120x + 64 = 0.$$

$S_1 : \Sigma\alpha = \alpha + \delta + \beta + \gamma = \dfrac{-2^{nd}}{1^{st}}$

$\alpha + \delta + \beta + \gamma = -15 \rightarrow$ (2) Reference by previous page

$S_2 : \Sigma\alpha\beta : (\alpha+\delta)(\beta+\gamma) + \alpha\delta + \beta\gamma = \dfrac{3^{rd}}{1^{st}}$

$(\alpha+\delta)(\beta+\gamma) + \alpha\delta + \beta\gamma = 70$ — (3)

$S_4 : \alpha\beta\gamma\delta = \dfrac{-4^{th}}{1^{st}}$

$(\alpha\delta)^2 = 64.$

$\alpha\delta = 8$ —— (4)

Sub (4) in (3)

$(\alpha+\delta)(\beta+\gamma) + 2 \times 18 = 70$

$(\alpha+\delta)(\beta+\gamma) = 70 - 36$

$(\alpha+\delta)(\beta+\gamma) = 54.$ —— (5)

Consider :-

$(\alpha+\delta) - (\beta+\gamma) = \sqrt{(\alpha+\delta+\beta+\gamma)^2 - 4(\alpha+\delta)(\beta+\gamma)}$

$= \sqrt{(-15)^2 - 4(54}$

$= \sqrt{225 - 216}$

$(\alpha+\delta) - (\beta+\gamma) = \sqrt{9} = 3$ —— (6)

(5) + (2) $(\alpha + \delta) - (\beta + \gamma) = 3$

$(\alpha + \delta) + (\beta + \gamma) = -15$

$$2(\alpha + \delta) = -12$$

$$\alpha + \delta = -6$$

(5) - (2)

$$2(\beta + \gamma) = 18$$

$$\beta + \gamma) = 9$$

Take $\alpha\delta = 8$; $\alpha + \delta = -6$

$x^2 - (\text{sum of the roots})x + \text{Product of the roots} = 0$

$$x^2 + 6x + 8 = 0$$

$$(x+4)(x+2) = 0$$

$$\boxed{x = -2, -4}$$

Take $\beta + \gamma = 9$ $\beta\gamma = 8$

$$x^2 + 9x + 8 = 0$$

$$(x+1)(x+8) = 0$$

$$x = -1, -8$$

$8 < {+1 \atop +8}$; $+9$

$\therefore$ the roots are $-4, -2, -1, -8$

(3) $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$.

Solution :-

Let the four roots are $\alpha, \beta, \gamma, \delta$

$$\alpha\delta = \beta\gamma$$

$$S_1 = \Sigma\alpha = \alpha + \delta + \beta + \gamma = \frac{-2^{nd}\text{ coef}}{1^{st}\text{ coef}}$$

$$(\alpha+\delta) + (\beta+\gamma) = \frac{40}{3} \quad \text{--- (1)}$$

$$S_2 : \Sigma\alpha\beta = (\alpha+\delta)+(\beta+\gamma)$$

$$+\alpha\delta + \beta\gamma = \frac{3^{rd}\text{ coef}}{1^{st}\text{ coef}}$$

$$(\alpha+\delta)+(\beta+\gamma) + \alpha\delta + \beta\gamma = \frac{130}{3} \quad \text{--- (2)}$$

$$S_4 : \alpha\beta\gamma\delta = \frac{-5^{th}\text{ coef}}{1^{st}\text{ coef}} =$$

$$(\alpha\delta)^2 = \cancel{120/3}\ 40\ \ 27/3$$

$$(\alpha\delta)^2 = \cancel{40}\ \ 9$$

$$\boxed{\alpha\delta = \cancel{20}\,3} \quad \text{--- (3)}$$

120

Sub (3) in (2).

$$(\alpha+\delta)(\beta+\gamma) + 2(\cancel{20}\,3) = \frac{130}{3}$$

$$(\alpha+\delta)(\beta+\gamma) = \frac{130}{3} - \cancel{40}\,6$$

$$= \frac{130 - \cancel{120}\,18}{3}$$

$$(\alpha+\delta)(\beta+\gamma) = \frac{10\cancel{2}}{3} = \frac{112}{3}$$

Consider :-

$$(\alpha+\delta)-(\beta+\gamma) = \sqrt{(\alpha+\delta+\beta+\gamma)^2 - 4(\alpha+\delta)(\beta+\gamma)}$$

$$= \sqrt{\left(\frac{40}{3}\right)^2 - 4\left(\frac{112}{3}\right)}$$

$$\begin{array}{r} 112 \times 4 \\ \hline 448 \end{array}$$

$$= \sqrt{\frac{1600}{9} - \frac{448}{3}}$$

$$\begin{array}{r} 448 \times 3 \\ \hline 1344 \end{array}$$

$$= \sqrt{\frac{1600 - 1344}{9}}$$

$$\begin{array}{r} 1600 \\ 1344 \\ \hline 256 \end{array}$$

$$= \sqrt{\frac{256}{9}} \quad \frac{16}{3} \quad = \frac{16}{3}$$

$$= \cancel{\sqrt{8}}$$

$$= \cancel{\sqrt{\frac{16}{3}}}$$

$$(\alpha+\delta)-(\beta+\gamma) = \frac{16}{3} \quad — ⑤$$

⑤ + ④

$$(\alpha+\delta)+(\beta+\gamma) = \frac{40}{3}$$

$$(\alpha+\delta)-(\beta+\gamma) = \frac{16}{3}$$

(−)

$$2(\alpha+\delta) = \frac{56}{3}$$

$$\alpha+\delta = \frac{\cancel{56}^{28}}{\cancel{6}\,3}$$

$$\alpha+\delta = \frac{28}{3}$$

$$\begin{array}{r} 40 \\ -16 \\ \hline 24 \end{array}$$

⑤ − ①

$$2(\beta+\gamma) = \frac{24}{3}$$

$$(\beta+\gamma) = \frac{24}{6} = 4$$

$x^2 - (\text{sum of the roots})x + \text{Product of roots} = 0$

Take $\alpha\delta = 3$ $\quad \alpha + \delta = \frac{28}{3}$

$$x^2 - \frac{28}{3}x + 3 = 0 .$$

$\times 3$

$$3x^2 - 28x + 9 = 0$$

$27 \begin{cases} -1/3 \\ -27/3 \; 9 \end{cases}$ $\quad -28$

$$(3x-1)(x-9) = 0$$

$$x = 1/3 \quad x = 9 .$$

Take $\beta\gamma = 3$ $\quad \beta + \gamma = 4 .$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$3 \begin{cases} -1 \\ -3 \end{cases}$ $\quad -4$

$$x = 1 \quad x = 3 .$$

The roots are $1/3, 1, 3, 9$ //

1.7.19 The three roots are in H.P

1 Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in H.P [D]

Solution :

Given : the roots are in H.P

Let the roots are α, β, γ

Let $\frac{2}{\beta} = \frac{1}{\alpha} + \frac{1}{\gamma}$

$$\Rightarrow \frac{2}{\beta} = \frac{\alpha + \gamma}{\alpha\gamma}$$

$$\Rightarrow 2\alpha\gamma = \alpha\beta + \beta\gamma \quad \text{——— (2)}$$

$$S_1 = \Sigma\alpha = \alpha + \beta + \gamma = \frac{-2^{nd}\text{ coef}}{1^{st}\text{ coef}}$$

$$= \frac{18}{81}$$

$$\alpha + \beta + \gamma = \frac{2}{9} \quad \text{——— (1)}$$

$$S_2 = \Sigma\alpha\beta = \alpha\beta + \alpha\gamma + \beta\gamma = \frac{3^{rd}\text{ coef}}{1^{st}\text{ coef}}$$

$$\alpha\gamma + 2\alpha\gamma = \frac{-36}{81} \quad [\text{by (2)}]$$

$$3\alpha\gamma = \frac{-4}{9}$$

$$\alpha\gamma = \frac{-4}{27} \quad \text{——— (3)}$$

$$S_3 = \Sigma\alpha\beta\gamma = \alpha\beta\gamma = \frac{-4^{th}\text{ coef}}{1^{st}\text{ coef}}$$

$$\frac{-4}{27}\beta = \frac{-8}{81} \qquad \beta = \frac{\cancel{-8}^{\,2}}{\cancel{81}_{\,\cancel{9}\,3}} \times \frac{\cancel{27}^{\,3}}{\cancel{-4}} \quad (\text{by (3)})$$

$$\boxed{\beta = \frac{2}{3}}$$

$$\boxed{\beta = \frac{2}{3}}$$

sub $\beta = \frac{2}{3}$ in (I)

$$\alpha + \beta + \gamma = \frac{2}{9}$$

$$\alpha + \frac{2}{3} + \gamma = \frac{2}{9}$$

$$\alpha + \gamma = \frac{2}{9} - \frac{2}{3}$$

$$= \frac{2-6}{9}$$

$$\boxed{\alpha + \gamma = \frac{-4}{9}}$$

We know that $\alpha + \gamma = -\frac{4}{9}$ $\alpha\gamma = \frac{-4}{27}$

x^2 - (sum of the roots) x + product of the roots $= 0$

$$x^2 + \frac{4}{9}x - \frac{4}{27} = 0$$

$\times 27$ $$27x^2 + 12x - 4 = 0$$

$a = 2\cancel{4}$ $b = 12$ $c = -4$.

$$24 \times 24 = 576$$

$$b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = \frac{-12 \pm \sqrt{144 - 4(2\cancel{4})(-4)}}{4(24)}$$

$$= \frac{-12 \pm \sqrt{144 + \cancel{432}}}{96}$$

$$= \frac{-12 \pm \sqrt{\cancel{528}}\ \sqrt{576}}{54}$$

$$b = \frac{-12 \pm 24}{54} = \cancel{\frac{-12}{54}} \cancel{\pm 24}$$

$b = -36/54$ or $12/54$ $x = -\frac{2}{3}$ and $\frac{2}{9}$

$\therefore$ The roots are $\frac{-2}{3}, \frac{2}{3}, \frac{2}{9}$

2 $3x^3 + 11x^2 + 12x + 4 = 0$ — (1) whose roots are in H.P

Solution :-

Given :- The roots are in H.P

Let the roots are α, β, γ

Let $\frac{2}{\beta} = \frac{1}{\alpha} + \frac{1}{\gamma}$

$$\frac{2}{\beta} = \frac{\alpha + \gamma}{\alpha\gamma}$$

$$\Rightarrow 2\alpha\gamma = \alpha\beta + \beta\gamma \quad \text{——— (2)}$$

$$S_1 = \Sigma\alpha = \alpha + \beta + \gamma = \frac{-2^{nd}\ \text{coef}}{1^{st}\ \text{coef}}$$

$$\alpha + \beta + \gamma = \frac{-11}{3} \quad \text{——— (3)}$$

$$S_2 = \Sigma\alpha\beta = \alpha\cdot\beta + \alpha\gamma + \beta\gamma = \frac{3^{rd}\ \text{coef}}{1^{st}\ \text{coef}}$$

$$\alpha\gamma + 2\alpha\gamma = \frac{12}{3} \quad [\text{by } (2)]$$

$$3\alpha\gamma = 4$$

$$\alpha\gamma = \frac{4}{3} \quad \text{——— (4)}$$

$$S_3 = \Sigma\alpha\beta\gamma = \alpha\beta\gamma = \frac{-4^{th}\ \text{coef}}{1^{st}\ \text{coef}}$$

$$\frac{4}{3}\beta = -\frac{4}{3}$$

$$\beta = -\frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}}$$

$$\boxed{\beta = -1}$$

Sub $\beta = -1$ in (3)

$$\alpha + \beta + \gamma = -11/3$$

$$\alpha - 1 + \gamma = -11/3$$

$$\alpha + \gamma = -11/3 + 1$$

$$= \frac{-11+3}{3}$$

$$\boxed{\alpha + \gamma = -8/3}$$

We know that

$$\alpha\gamma = \frac{4}{3} \qquad \alpha + \gamma = -8/3$$

$x^2 -$ (sum of ro) x + prod of the r $= 0$

$$x^2 + \frac{8}{3}x + \frac{4}{3} = 0$$

×3

$$3x^2 + 8x + 4 = 0$$

$$(x+2)(3x+2) = 0$$

$$12 \begin{cases} 2 \\ 6/3 \\ 2/3 \end{cases} \quad 8$$

$$x = -2 \text{ or } x = -\frac{2}{3}.$$

$$x = -2, -2/3$$

Therefore,

The roots are $-\frac{2}{3}, -2, -1$

1. Solve the equation.

$$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$$

Gn that 2 of its roots are equal in magnitude and are opposite in sign.

Sol $\alpha, \beta, \gamma, \delta$

Let $\alpha = -\beta \Rightarrow \alpha + \beta = 0$ —— (1)

$$S_1 = \Sigma\alpha : \alpha + \beta + \gamma + \delta = -\frac{2^{nd}\ coef}{1^{st}\ coef}$$

$$0 + \gamma + \delta = \frac{+2}{1}$$

$$\gamma + \delta = 2 \text{ ———— (2)}$$

$$S_2 = \Sigma\alpha\beta = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{3^{rd}\ coef}{1^{st}\ coef} \quad (\text{by } (1))$$

$$\delta(\alpha + \beta) + \gamma(\alpha + \beta) + \alpha\beta + \gamma\delta = \frac{4}{1}$$

$$(\alpha + \beta)(\delta + \gamma) + \alpha\beta + \gamma\delta = 4$$

$$\alpha\beta + \gamma\delta = 4 \text{ — (3)}$$

$$S_3 = \Sigma\alpha\beta\gamma = \alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = -\frac{4^{th}\ cof}{1^{st}\ cof} \quad (\because \text{by } (1))$$

$$\alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) = -6.$$

$$\alpha\beta(\gamma + \delta) = -6$$

$$\alpha\beta(2) = -6$$

$$\alpha\beta = -3 \ (\text{by } (3))$$

$$S_4 = \Sigma\alpha\beta\gamma\delta \quad \alpha\beta\gamma\delta = \frac{5^{th}\ coff}{1^{st}\ coff}$$

$$-3\gamma\delta = -21 \quad \boxed{\gamma\delta = 7}$$

Sub (4) in (3) $\alpha\beta = -3$ in (3)

$$-3 + \gamma\delta = 4$$

$$\gamma\delta = 4 + 3$$

$$\boxed{\gamma\delta = 7}$$

WKT

$x^2 -$ (sum of the r) x + product of the roots $= 0$

$\alpha\beta = -3$ $\quad x^2 - 0x + 3 = 0.$ $\quad \alpha + \beta = 0$; $\alpha\beta =$

$\alpha + \beta = 0$ $\quad x = \pm\sqrt{3}.$

WKT

$$x^2 - (\text{sum})x + \text{pro} = 0.$$

$$\gamma + \delta = 2 \; ; \; \gamma\delta = 7$$

$$x^2 - 2x + 7 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(7)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 28}}{2}$$

$$= \frac{2 \pm \sqrt{-24}}{2} = \frac{2 \pm \sqrt{(-1) \times 2 \times 2 \times 2 \times 3}}{2}$$

$$x = 1 \pm i\sqrt{6}$$

(2) The sum of the two roots of the equation

$$x^4 + px^3 + qx^2 + rx + s = 0$$

Equals the sum of the other two. Prove that $p^3 + 8r = 4pq$

Solution

Let the four roots are $\alpha, \beta, \gamma, \delta$

Gn:- $\boxed{\alpha + \beta = \gamma + \delta}$

$S_1 :- \Sigma\alpha : \alpha + \beta + \gamma + \delta = \dfrac{-2^{nd} \text{ coefficient}}{1^{st} \text{ coefficient}}$

$$(\alpha+\beta) + (\alpha+\beta) = \frac{-p}{1}$$

$$2(\alpha+\beta) = -p$$

$$\alpha + \beta = -p/2 \quad \text{———} \quad (1).$$

$S_2 :- \Sigma\alpha\beta \quad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \dfrac{+3^{rd} \text{ coeff}}{1^{st} \text{ coeff}}$

$$\alpha(\gamma+\delta) + \beta(\gamma+\delta) + \alpha\beta + \gamma\delta = q$$

$$(\alpha+\beta)(\gamma+\delta) + \alpha\beta + \gamma\delta = q$$

$$(\alpha+\beta)^2 + \alpha\beta + \gamma\delta = q$$

$$\left(\frac{-p}{2}\right)^2 + \alpha\beta + \gamma\delta = q$$

$$\alpha\beta + \gamma\delta = q - \frac{p^2}{4}$$

$S_3 = \Sigma\alpha\beta\gamma =$

$$\alpha\beta\gamma + \cancel{\alpha\gamma\delta} + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = \frac{-4^{th}}{1^{st}}$$

$$\alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = -r$$

$$\alpha\beta(\alpha+\beta) + \gamma\delta(\alpha+\beta) = -r$$

$$(\alpha+\beta)(\alpha\beta+\gamma\delta) = -r$$

$$\left(\frac{-p}{2}\right)\left(q - \frac{p^2}{4}\right) = -r$$

$$\left(\frac{-p}{2}\right)\left(\frac{4q-p^2}{4}\right) = -r$$

$$\frac{-4pq+p^3}{8} = -r$$

$$-4pq+p^3 = -8r$$

$$-4pq+p^3+8r = 0$$

$$p^3+8r = 4pq$$

$$\therefore \quad p^3+8r = 4pq$$

Hence it is proved. The sum of the two roots equals of the sum of the other two roots.

Hint :-

The sum of the two roots of the equation equals the sum of the other two roots $\Rightarrow$ this Means $\boxed{\alpha+\beta = \gamma+\delta}$

Two of its roots are equal in magnitude and opposite in sign.

$\Rightarrow$ This means $\boxed{\alpha = -\beta \quad \text{or} \quad \gamma = -\delta}$

$$\boxed{\alpha+\beta = 0 \quad \text{or} \quad \gamma+\delta = 0}$$

(3) Show that the roots of the equation $x^3+px^2+qx+r=0$ are in A.P. If $2p^3-9pq+27r=0$. Show that the above condition is satisify by the equation $x^3-6x^2+13x-10=0$. Hence or otherwise solve the equation.

<u>Solution</u> :- $x^3+px^2+qx+r=0$.

Let the 3 roots are $(a-d), a, (a+d)$

<u>Gn</u>:-

$S_1 = \Sigma\alpha = a-\cancel{d}+a+a+\cancel{d} = \dfrac{-2^{th}\ \text{coeffi}}{1^{st}\ \text{coeffi}}$

$3a = -p$

$\boxed{a = -\dfrac{p}{3}}$ —— (1)

$S_2 = \Sigma\alpha\beta = \cancel{6a^2 \ \ lot^2} = \dfrac{3^{rd}\ \text{coef}}{1^{st}\ \text{coef}}$

$(a-d)a + a(a+d) + (a-d)(a+d)$

$a^2-ad+a^2+ad+a^2-d^2 = q$

$3a^2-d^2 = q$

$\dfrac{3p^2}{9_3} - d^2 = q$

$\dfrac{p^2}{3} - d^2 = q$

$-d^2 = q - \dfrac{p^2}{3}$

$(\times-)$ $\boxed{d^2 = \dfrac{p^2}{3} - q}$ —— (2)

$S_3 : \Sigma\alpha\beta\gamma : (a-d)(a)(a+d) = \frac{-4^{th} \text{ coef}}{1^{st} \text{ co-ef}}$

$$(a^2 - d^2)a = -r$$

$$a^3 - ad^2 = -r \quad \text{——} \quad (3)$$

Sub (3) in (1) and (2).

$$\frac{-p}{27} - \left(\frac{-p}{3}\right)\left(\frac{p^2}{3} - q\right) = -r$$

$$\frac{-p}{27} - \left(\frac{-p}{3}\right)\left(\frac{p^2 - 3q}{3}\right) = -r$$

$$\frac{-p}{27} - \left(\frac{-p^3 + 3pq}{9}\right) = -r$$

$$\frac{-9p - (27p^3 + 81pq}{27} = -r$$

$27 \times 3 = 81$

$$-9p + 27p^3 - 81pq = -27r$$

$$2p^3 + 27r = 9pq \quad \text{——} \quad (4)$$

In the equation, $x^3 - 6x^2 + 13x - 10 = 0$

there, Compare this equation with.

$$x^3 + px^2 + qx + r = 0$$

$$p = -6 \quad q = 13 \quad r = -10.$$

(4) becomes

$$2(-6)^3 + 27(-10) = 9(13)(-6).$$

$$-2(216) + (-270) = -702$$

$36 \times 6 = 216$

$$-432 - 270 = -702$$

$-702 + 702 = 0$

Therefore the condition is satisfied

So, the roots of the equation are in A.P

$$3a = -p$$

$$3a = +6$$

$$\boxed{a = 2}$$

(2) $$d^2 = \frac{36}{3} - 13$$

$$= \frac{36 - 39}{3}$$

$$= \frac{-3}{3}$$

$$d^2 = -1.$$

$$d = \pm i$$

Therefore, the roots are $2-i, 2, 2+i$.

4) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometrical Progression.

Solve the eq $27x^3 + 42x^2 - 28x - 8 = 0$ whose roots are in G.P.

Let the 3 roots are $\frac{k}{r}, k, kr$

Solution :-

$S_1 : \Sigma\alpha : \frac{k}{r} + k + kr = \frac{-2^{nd} \text{ coef}}{1^{st} \text{ coef}}$

$$\frac{k + kr + kr^2}{r} = -\frac{42}{27} \; -\frac{3b}{a}$$

$$k\left(\frac{1}{r} + 1 + r\right) = -\frac{3b}{a} \text{ —— (1)}$$

$S_2 : \Sigma\alpha\beta \quad \cancel{\frac{k}{r}}k + (k)(kr) + (kr)\left(\frac{k}{r}\right) = \frac{3^{rd} \text{ coef}}{1^{st} \text{ coef}}$

$$\frac{k^2}{r} + k^2 r + k^2 = \frac{3c}{a}$$

$$k^2\left(\frac{1}{r} + r + 1\right) = \frac{3c}{a} \text{ — (2)}$$

$S_3 : \Sigma\alpha\beta\gamma \quad \left(\frac{k}{r}\right)(k)(kr) = \frac{-4^{th} \text{ coef}}{1^{st} \text{ coef}}$

$$k^3 = -\frac{d}{a} \text{ —— (3)}$$

Eq $\frac{(2)}{(1)}$.

$$\frac{k^2\left(\cancel{\frac{1}{r} + r + 1}\right)}{k\left(\cancel{\frac{1}{r} + r + 1}\right)} = \frac{3c}{\cancel{a}} \times \frac{\cancel{a}}{-3b}$$

$$\boxed{k = -\frac{c}{b}} \text{ ------ (4)}$$

Sub (4) in (3)

$$\left(\frac{-c}{b}\right)^3 = -\frac{d}{a}$$

$$\frac{c^3}{b^3} = \frac{d}{a}$$

$$c^3 a = b^3 d$$

Solve the equation

$$27x^3 + 42x^2 - 28x - 8 = 0.$$

Therefore Eq ①. becomes.

$$k\left(\frac{1}{r} + 1 + r\right) = \frac{-42}{27} \quad \text{—— ④}$$

Therefore Eq ② becomes

$$k^2\left(\frac{1}{r} + r + 1\right) = \frac{-28}{27}. \quad \text{—— ⑤}$$

therefore Eq ③ becomes

$$k^3 = 8/27$$

$$\boxed{\cancel{k = \frac{2}{2}}} \quad k^3 = \frac{8}{27}$$

$$\boxed{k = \frac{2}{3}} \quad \text{—— ⑥}$$

Sub ⑥ ~~in~~ ⑤/④

$$\frac{k^2(1/r + r + 1)}{k(1/r + r + 1)} = \frac{-28}{27} \times \frac{27}{-42}$$

$$\boxed{k = 2/3}$$

Sub ⑥ in $k = 2/3$ in ④

$$\frac{2}{3}\left(\frac{1}{r} + 1 + r\right) = \frac{-42}{27}$$

$$\frac{1}{r} + 1 + r = \frac{-42}{27\,9} \times \frac{3}{2}$$

$$r + r^2 + 1 = \frac{-21r}{9}$$

$$9r^2 + 9r + 9 = -21r$$

$$9r^2+9r+21r+9=0$$

$$9r^2+30r+9=0.$$

$\div 3$ $$3r^2+10r+3=0.$$ $9 < \frac{-1}{-9}$ 10

$$(r+3)(3r+1)=0$$

$\boxed{r=3}$ $\boxed{r=-1/3}$

The roots are $k/r, k, kr$.

$$= -2, 2/3, -\frac{2}{9}$$

$\therefore$ The roots are $-2, -2/9, 2/3$

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1 Sum of the powers of the roots of the equation. (3)

Solution :-

Let $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_n$ be the roots of the equation $f(x)=0$.

The sum of r^{th} powers of the roots

ie. $\alpha_1^r+\alpha_2^r+\ldots+\alpha_n^r$

and is usually denoted by S_r. There are two methods can be used to find S_r.

METHOD - 1 (3)

S_r = coefficient of $\frac{1}{x^r}$ in the expansion of $\frac{xf'(x)}{f(x)}$

Find the sum of cubes of the roots of equation $x^5 = x^2 + x + 1$.

Solution

Gn $x^5 = x^2 + x + 1$

$\Rightarrow x^5 - x^2 - x - 1 = 0$

$f(x) = x^5 - x^2 - x - 1$

$f'(x) = 5x^4 - 2x - 1$

$xf'(x) = 5x^5 - 2x^2 - x$

WKT.

S_r = coefficient of $\frac{1}{x^r}$ in the expansion of $\frac{xf'(x)}{f(x)}$

$\therefore$ S_3 = coefficiend of $\frac{1}{x^3}$ in the exp of $\frac{xf'(x)}{f(x)}$

$$= \frac{5x^5 - 2x^2 - x}{x^5 - x^2 - x - 1}$$

$$= \frac{x^5\left(5 - \frac{2}{x^3} - \frac{1}{x^4}\right)}{x^5\left(1 - \frac{1}{x^3} - \frac{1}{x^4} - \frac{1}{x^5}\right)}$$

$$= \frac{5 - \frac{2}{x^3} - \frac{1}{x^4}}{1 - \frac{1}{x^3} - \frac{1}{x^4} - \frac{1}{x^5}}$$

$$= \frac{5 - \frac{2}{x^3} - \frac{1}{x^4}}{1 - \left(\frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right)}$$

$= \quad '' \quad \left(5 - \frac{2}{x^3} - \frac{1}{x^4}\right)\left(1 - \left(\frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right)\right)^{-1}$

$= \quad '' \quad \left(5 - \frac{2}{x^3} - \frac{1}{x^4}\right)\left(1 + \left(\frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right) + \left(\frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right)^2\right)$

$\because \left[(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots\right]$

$= \quad '' \quad '' \quad \left(5 - \frac{2}{x^3} - \frac{1}{x^4}\right)\left(1 + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5}\right)$

$= \quad '' \quad '' \quad -\frac{2}{x^3} + \frac{5}{x^3}$

$=$ coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{3}{x^3}$

$= 3.$

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1) Calculate the sum of cubes of roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$

Solution :-

S_3 = coef of $\frac{1}{x^3}$ of the expansion of $\frac{x f'(x)}{f(x)}$

$= \quad '' \quad \frac{x(3x^2 - 12x + 11)}{x^3 - 6x^2 + 11x - 6}$

$= \quad '' \quad \frac{3x^3 - 12x^2 + 11x}{x^3 - 6x^2 + 11x - 6}$

$$= \text{''} \quad \text{''} \quad \frac{\cancel{x^3}\left(3-\frac{12}{x}+\frac{11}{x^2}\right)}{\cancel{x^3}\left(1-\frac{6}{x}+\frac{11}{x^2}-\frac{6}{x^3}\right)}$$

$$= \text{''} \quad \left(3-\frac{12}{x}+\frac{11}{x^2}\right)\left(1-\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)\right)^{-1}$$

$$= \frac{\left(3-\frac{12}{x}+\frac{11}{x^2}\right)}{1-\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)}$$

$$= \left(3-\frac{12}{x}+\frac{11}{x^2}\right)\left(1-\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)\right)^{-1}$$

$$= \left(3-\frac{12}{x}+\frac{11}{x^2}\right)\left(1+\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)+\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)^2 + \left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)^3\right)$$

$$= \left(3-\frac{12}{x}+\frac{11}{x^2}\right)\left(1+\left(\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}\right)+\left(\frac{6}{x}\right)^2 + 2\left(\frac{6}{x}\right)\left(\frac{-11}{x^2}\right)+\left(\frac{6}{x}\right)^3\right)$$

$$= \text{''} \quad \left(3-\frac{12}{x}+\frac{11}{x^2}\right)\left(1+\frac{6}{x}-\frac{11}{x^2}-\frac{6}{x^3}+\frac{36}{x^2}-\frac{132}{x^3}+\frac{216}{x^3}\right]$$

$$= \quad " \quad " \left(3 - \frac{12}{x} + \frac{11}{x^2}\right)\left(1 + \frac{6}{x} + \frac{25}{x^2} + \frac{90}{x^3}\right)$$

$$= \quad " \quad " \left(\frac{3 \times 90}{x^3} - \frac{12 \times 25}{x^3} + \frac{66}{x^3}\right)$$

$$= \left(\frac{+270 - 300 - 66}{x^3}\right)$$

$$= \frac{336 - 300}{x^3}$$

$$= 36$$

---

2) Calcute the sum of cubes of roots of the equation

(i) $x^4 + 2x + 3 = 0$ (ii) $x^3 - 5x^2 - 16x + 80 = 0$

(ii) WKT.

$$f(x) = x^4 + 2x + 3$$

$$f'(x) = 4x^3 + 2$$

$$xf'(x) = 4x^4 + 2x$$

WKT.

S_3 = coef of $\frac{1}{x^3}$ in the expansion of $\frac{xf'(x)}{f(x)}$

$= \quad " \quad " \quad \dfrac{4x^4+2x}{x^4+2x+3}$

$= \quad " \quad " \quad \dfrac{x^4\left(4+\frac{2}{x^3}\right)}{x^4\left(1+\frac{2}{x^3}+\frac{3}{x^4}\right)}$

$= \quad " \quad " \quad \dfrac{\left(4+\frac{2}{x^3}\right)}{1-\left(-\frac{2}{x^3}-\frac{3}{x^4}\right)}$

$= \left(4+\frac{2}{x^3}\right)\left(1-\left(\frac{-2}{x^3}-\frac{3}{x^4}\right)\right)^{-1}$

$= \left(4+\frac{2}{x^3}\right)\left(1+\left(\frac{-2}{x^3}-\frac{3}{x^4}\right)+\left(\frac{-2}{x^3}-\frac{3}{x^4}\right)^2\right.$

$= \left(4+\frac{2}{x^3}\right)\left(1+\frac{-2}{x^3}-\frac{2}{x^4}\right)$

$= \dfrac{-2\times 4}{x^3}+\dfrac{2}{x^3}$

$= \dfrac{-8}{x^3}+\dfrac{2}{x^3}$

= coefficient of $\frac{1}{x^3}$ in the exp of $\frac{-6}{x^3}$

$\boxed{S_3 = -6}$ //

(ii) $x^3 - 5x^2 - 16x + 80 = 0$.

Sol

$$f(x) = x^3 - 5x^2 - 16x + 80$$

$$f'(x) = 3x^2 - 10x - 16$$

$$xf'(x) = 3x^3 - 10x^2 - 16x$$

WKT

S_3 = coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{xf'(x)}{f(x)}$

$$= \text{〃} \qquad \text{〃} \qquad \text{〃} \qquad \frac{3x^3 - 10x^2 - 16x}{x^3 - 5x^2 - 16x + 80}$$

$$= \frac{\cancel{x^3}\left(3 - \frac{10}{x} - \frac{16}{x^2}\right)}{\cancel{x^3}\left(1 - \frac{5}{x} - \frac{16}{x^2} + \frac{80}{x^3}\right)}$$

$$= \frac{\left(3 - \frac{10}{x} - \frac{16}{x^2}\right)}{1 - \left(+\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)}$$

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 - \left(+\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)\right)^{-1}$$

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 + \left(\frac{+5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right) + \left(\frac{+5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)^2\right.$$

$$\left. + \left(\frac{+5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)^3\right.$$

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left[\left(1 - \left(\frac{+5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right) + \left(\frac{+5}{x}\right)^2 + 2\left(\frac{-5}{x}\right)\left(\frac{16}{x^2}\right) + \left(\frac{-5}{x}\right)^3\right]$$

160
125
20
365

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 + \frac{5}{x} - \frac{16}{x^2} + \frac{80}{x^3} + \frac{25}{x^2} + \frac{160}{x^3} + \frac{125}{x^3}\right)$$

$$= \left(\frac{3\times 80}{x^3} + \frac{125\times 3}{x^3} + \frac{10\times 16}{x^3} - \frac{25\times 10}{x^3} + \frac{16\times 5}{x^3}\right)$$

365×3
1095

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 + \frac{5}{x} + \frac{9}{x^2} + \frac{365}{x^3}\right)$$

16
135
35

$$= \left(\frac{3\times 365}{x^3} - \frac{90}{x^3} - \frac{80}{x^3}\right) = \frac{1095 - 170}{x^3}$$

6
-170
45
-125

= coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{-35 \; \cancel{-125}}{x^3}$

= coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{-35 \; \cancel{-125}}{x^3}$

~~= -125~~

$= -35$

$\boxed{S_3 = -35}$

Method - R. (4)

Newton's theorem on the sum of power of the roots

$$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots + rP_r = 0 \text{ If } r \le n.$$

$$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots + P_n S_{r-n} = 0 \text{ If } r > n.$$

Find the sum of 5^{th} power of the roots of the equation $x^4 - 7x^2 - 4x - 3 = 0$

Gin $\quad x^4 - 7x^2 - 4x - 3 = 0$ —— (1)

$r = 5$; $n = 4$.

$P_1 = 1 \quad P_2 = 0, \quad P_3 = -7. \quad P_4 = -4 \quad P_5 = -3.$

$r > n \quad 5 > 4.$

$S_5 + P_1 S_4 + P_2 S_3 + P_3 S_2 + P_4 S_1 = 0.$

$S_5 + S_4 - 7S_2 - 4S_1 = 0.$

$S_5 = -S_4 + 7S_2 + 4S_1$

1). $\quad x^4 - 7x^2 - 4x - 3 = 0$

Here $n = 4$; $r = 5 \quad r > n$.

$P_1 = 0$; $P_2 = -7 \quad P_3 = -4 \quad P_4 = -3.$

$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots P_n P_r = 0$ If $r > n$.

$S_5 + P_1 S_4 + P_2 S_3 + P_3 S_2 + P_4 S_1 = 0$

$S_5 - 7S_3 - 4S_2 - 3S_1 = 0$

$$S_5 = 7S_3 + 4S_2 + 3S_1 \quad \text{--- (1)}$$

To find S_3 : Here $r = 3$ $n = 4$

$$S_3 + P_1 S_2 + P_2 S_1 + 3P_3 = 0.$$

$$S_3 + 0 - 7S_1 - 12 = 0$$

$$S_3 = 7S_1 + 12 \quad \text{--- (2)}$$

To find S_2 : Here $r = 2$ $n = 4$

$$S_2 + P_1 S_1 + 2P_2 = 0.$$

$$S_2 - 14 = 0$$

$$\boxed{S_2 = 14} \quad \text{--- (3)}$$

To find S_1 Here $r = 1$ $n = 4$.

$$S_1 + P_1 S_0 + \cancel{P_2 S_{<1}} 1P_1 = 0$$

$$S_1 + 1(0) = 0.$$

$$\boxed{S_1 = 0}$$

$12 \times 7 = 84$

∴ (2) becomes.

$$\boxed{S_3 = 12}$$

∴ (1) becomes,

$$S_5 = 7 \times 12 + 4 \times 14 + 0$$

$$= 84 + 56$$

$$= 140$$

1 Find the 11th power of the equation

$x^7 + 5x^4 + 1 = 0.$ $S_{11} = 0$

Solution :-

Here $r = 11$ $n = 7$

$P_1 = 0$ $P_2 = 0$ $P_3 = 5$ $P_4 = 0$ $P_5 = 0$ $P_6 = 0$ $P_7 = 1$

$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots\dots + P_n S_{r-n} = 0$ If $r > n$

$S_{11} + 0 S_{10} + 0 S_9 + 5 S_8 + 0 S_7 + 0 S_6 + 0 S_5 + S_4 = 0$

$S_{11} + 5S_8 + S_4 = 0.$

$S_{11} = -5S_8 - S_4.$ ——— (1).

To find S_8,

Here $r = 8$ $n = 7$

$S_8 + 0S_7 + 0S_6 + 5S_5 + 0S_4 + 0S_3 + 0S_2 + P_7 S_1 = 0$

$S_8 + 5S_5 + S_1 = 0.$

$S_8 = -5S_5 - S_1$ ——— (2).

To Find S_5

Here $r = 5$ $n = 7$.

$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots\dots + rP_r = 0$ If $r \leq n$

$S_5 + 0 S_4 + 0 S_3 + 5S_2 + 0S_1 + 5P_5 = 0.$

$S_5 + 5S_2 + 5(0) = 0.$

$S_5 + 5S_2 = 0.$

$S_5 = -5S_2$ ——— (3)

To find S_2,

Here $r=2$, $n=7$.

$S_2 + 0S_1 + 2P_2 = 0$.

$S_2 + 2(0) = 0$

$\boxed{S_2 = 0}$ —— ⊛

⊛ Sub $S_2 = 0$ in ③

$S_5 = -5(0)$

$\boxed{S_5 = 0}$.

To find S_1

Here $r=1$, $n=7$.

$S_1 + 0S_0 + 1P_1 = 0$.

$S_1 + 1(0) = 0$.

$\boxed{S_1 = 0}$

Sub $S_1 = 0$ in eq 5

To find S_4.

Here $n=7$ $r=4$.

$S_4 + 0S_3 + 0S_2 + 5S_1 + 4P_4 = 0$.

$S_4 + 5S_1 + 4(0) = 0$.

$S_4 = -5S_1$ —— ④.

Sub $S_1=0$ in equation ④

$S_4 = -5(0)$.

$\boxed{S_4 = 0}$. —— ⊛⊛

Sub $S_5 = 0$ and $S_1 = 0$ in equation ②

$S_8 = -5(0) - (0)$

$\boxed{S_8 = 0}$ —— ⊛⊛

⊛ Sub $S_4 = 0$ and $S_8 = 0$ in equation (1)

$$S_{11} = -5S_8 - S_4.$$

$$= -5(0) - 0$$

$$\boxed{S_{11} = 0}$$

(ii) $x^3 - 5x^2 - 16x + 80 = 0$

Solution :

$$f(x) = x^3 - 5x^2 - 16x + 80.$$

$$f'(x) = 3x^2 - 10x - 16.$$

$$xf'(x) = 3x^3 - 10x^2 - 16x$$

WKT

S_3 = Coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{xf'(x)}{f(x)}$

$$= \quad " \quad \frac{3x^3 - 10x^2 - 16x}{x^3 - 5x^2 - 16x + 80}$$

$$= \quad " \quad \frac{\cancel{x^3}\left(3 - \frac{10}{x} - \frac{16}{x^2}\right)}{\cancel{x^3}\left(1 - 5/x - \frac{16}{x^2} + \frac{80}{x^3}\right)}$$

$$= \quad " \quad \frac{\left(3 - \frac{10}{x} - \frac{16}{x^2}\right)}{1 - \left(5/x + 16/x^2 - \frac{80}{x^3}\right)}$$

$$= \quad " \quad \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 - \left(\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)\right)^{-1}$$

$$= \left(3 - \frac{10}{x} - \frac{16}{x^2}\right)\left(1 + \left(\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right) + \left(\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)^2\right.$$

$$\left. + \left(\frac{5}{x} + \frac{16}{x^2} - \frac{80}{x^3}\right)^3\right)$$

$$=\left(3-\frac{10}{x}-\frac{16}{x^2}\right)\left(1+\left(\frac{5}{x}+\frac{16}{x^2}-\frac{80}{x^3}\right)+\left(\frac{5}{x}+\frac{16}{x^2}-\frac{80}{x^3}\right)^2+\left(\frac{5}{x}+\frac{16}{x^2}-\frac{80}{x^3}\right)^3\right)$$

$$=\left(3-\frac{10}{x}-\frac{16}{x^2}\right)\left(1+\left(\frac{5}{x}+\frac{16}{x^2}-\frac{80}{x^3}\right)+\left(\frac{5}{x}\right)^2+2\left(\frac{5}{x}\right)\left(\frac{16}{x^2}\right)+\left(\frac{5}{x}\right)^3\right)$$

$$=\left(3-\frac{10}{x}-\frac{16}{x^2}\right)\left(1+\frac{5}{x}+\frac{16}{x^2}-\frac{80}{x^3}+\frac{25}{x^2}+\frac{160}{x^3}+\frac{125}{x^3}\right)$$

$$=\left(3-\frac{10}{x}-\frac{16}{x^2}\right)\left(1+\frac{5}{x}+\frac{16+25}{x^2}+\frac{-80+160+125}{x^3}\right)$$

$$=\left(3-\frac{10}{x}-\frac{16}{x^2}\right)\left(1+\frac{5}{x}+\frac{41}{x^2}+\frac{205}{x^3}\right)$$

$$=\left(\frac{3\times 205}{x^3}-\frac{16\times 5}{x^3}-\frac{41\times 10}{x^3}\right)$$

$$\begin{array}{r} 205\times 3 \\ \hline 615 \end{array}$$

$$=\left(\frac{615-80-410}{x^3}\right)$$

$$\begin{array}{r} 615 \\ 490 \\ \hline 125 \end{array}$$

$$=\left(\frac{615-490}{x^3}\right)$$

$$=\frac{125}{x^3}$$

= coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{125}{x^3}$

= coefficient of $\frac{1}{x^3}$ in the expansion of $\frac{125}{x^3}$ is 125

2. Find the 4^{th} power of the equation
$x^4 - 7x^2 - 4x - 3 = 0$.

Method - 1

Solution :-

Gn Equation :- $x^4 - 7x^2 - 4x - 3 = 0$.

WKT.

S_r = coefficient of $\frac{1}{x^r}$ in the expansion of $\frac{x f'(x)}{f(x)}$

$f(x) = x^4 - 7x^2 - 4x - 3$

$f'(x) = 4x^3 - 14x - 4$.

$x f'(x) = 4x^4 - 14x^2 - 4x$

S_4 = coefficient of $\frac{1}{x^4}$ in the expansion of $\frac{x f'(x)}{f(x)}$

= " $\frac{4x^4 - 14x^2 - 4x}{x^4 - 7x^2 - 4x - 3}$

= " $\frac{\cancel{x^4}(4 - 14/x^2 - 4/x^3)}{\cancel{x^4}(1 - 7/x^2 - 4/x^3 - 3/x^4)}$

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$$= \frac{\left(4 - 14/x^2 - 4/x^3\right)}{\left(1 - 7/x^2 - 4/x^3 - 3/x^4\right)}$$

$$= \frac{\left(4 - \frac{14}{x^2} - 4/x^3\right)}{\left(1 - \left(\frac{7}{x^2} + \frac{4}{x^3} + \frac{3}{x^4}\right)\right)}$$

$$= \left(4 - \frac{14}{x^2} - \frac{4}{x^3}\right)\left(1 - \left(\frac{7}{x^2} + \frac{4}{x^3} + \frac{3}{x^4}\right)\right)^{-1}$$

$$= \left(4 - \frac{14}{x^2} - \frac{4}{x^3}\right)\left(1 + \left(\frac{7}{x^2} + \cancel{\frac{4}{x^3}} + \frac{3}{x^4}\right) + \left(\frac{7}{x^2} + \frac{4}{x^3} + \frac{3}{x^4}\right)^2\right)$$

$$= \left(4 - \frac{14}{x^2} - \frac{4}{x^3}\right)\left(1 + \frac{7}{x^2} + \frac{3}{x^4} + \left(\frac{7}{x^2}\right)^2\right)$$

$$= \left(4 - \frac{14}{x^2} - \frac{4}{x^3}\right)\left(1 + \frac{7}{x^2} + \frac{3}{x^4} + \frac{49}{x^4}\right)$$

$$= \left(4 - \frac{14}{x^2} - \frac{4}{x^3}\right)\left(1 + \frac{7}{x^2} + \frac{52}{x^4}\right)$$

$$= \left(\frac{4 \times 52}{x^4} - \frac{14 \times 7}{x^4}\right)$$

$$= \left(\frac{208 - 98}{x^4}\right) = \frac{110}{x^4}$$

208
$-\ 98$
110

14×7
98

52×4
208
98
110

Find the sum of 9^{th} power of the equation $x^3+3x+9=0$

Solution :- $P_1=0$. $P_2=3$ $P_3=9$

WKT

Here $r=9$ $n=3$.

$$S_r + P_1 S_{r-1} + P_2 S_{r-2} + \dots\dots P_n S_{r-n} = 0 \text{ if } r>n$$

$$S_9 + 0S_8 + 3S_7 + 9S_6 \dots\dots + P_3 S_6 = 0$$

$$S_9 + 3S_7 + 9S_6 = 0.$$

$$S_9 = -3S_7 - 9S_6 \quad \text{——— (1).}$$

To find S_7

Here $r=7$ $n=3$.

$$S_7 + 0S_6 + 3S_5 + 9S_4 \dots\dots P_3 S_4 = 0$$

$$S_7 + 3S_5 + 9S_4 = 0. \quad \text{——— (2)}$$

$$S_7 = -3S_5 - 9S_4$$

To find S_5.

Here $r=5$ $n=3$.

$$S_5 + 0S_4 + 3S_3 + \dots\dots P_3 S_2 = 0$$

$$S_5 + 0S_4 + 3S_3 + 9S_2 = 0.$$

$$S_5 = -3S_3 - 9S_2 \quad \text{——— (3)}$$

To find S_3 $P_1 = 0$ $P_2 = 3$ $P_3 = 9$

Here $r = 3$ $n = 3$.

$S_r + P_1 S_{r-1} + P_2 S_{r-2} \cdots \cdots r P_r = 0$ If $r \leq n$

$S_3 + 0 S_2 + 3 S_1 + 3 P_3 = 0$.

$S_3 + 3 S_1 + 27 = 0$

$S_3 = -3 S_1 - 27$ ——— (4).

To find S_1

Here $r = 1$ $n = 3$.

$S_1 + 0 S_0 + 1 P_1 = 0$.

$\boxed{S_1 = 0}$ ——— (5).

sub $S_1 = 0$ in (4).

$S_3 = -3(0) - 27$

$\boxed{S_3 = -27}$

To find S_2.

Here $r = 2$ $n = 3$.

$S_2 + 0 S_1 + 3 P_0 + 2 P_2 = 0$.

$S_2 + 6 = 0$

$\boxed{S_2 = -6}$

To ~~fi~~ Sub $S_2 = -6$ and $S_3 = -27$ in (3)

$S_5 = -3 S_3 - 9 S_2$

$= -3(-27) - 9(-6)$

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$= 81 + 54$

$\boxed{S_5 = 135}$

$\begin{array}{r} \overset{2}{27} \times 3 \\ \hline 1 \end{array}$

To find S_4

Here $r = 4$ $n = 3$

$S_4 + 0S_3 + 3S_2 + \dots P_3 S_1 = 0.$

$S_4 + 3S_2 + 9S_1 = 0.$

$S_4 + 3(-6) + 9(0) = 0.$

$\boxed{S_4 = 18}$

Sub $S_4 = 18$ and $S_5 = 135$ in (2)

$S_7 = -3(135) - 9(18)$

$\begin{array}{r} 135 \times 3 \\ \hline 405 \end{array}$

$= -405 - 162$

$\begin{array}{r} 18 \times 9 \\ \hline 162 \end{array}$

$\boxed{S_7 = -567}$

To find S_6

Here $r = 6$ $n = 3$

$S_6 + 0S_5 + 3S_4 + \dots + P_3 S_3 = 0.$

$\begin{array}{r} 27 \times 9 \\ \hline 243 \end{array}$

$S_6 + 3S_4 + 9(-27) = 0$

$S_6 + 3(18) - 243 = 0.$

$S_6 + 54 - 243 = 0$

$\begin{array}{r} 18 \times 3 \\ \hline 54 \end{array}$

$\begin{array}{r} 243 \\ -54 \\ \hline 189 \end{array}$

$S_6 = 243 - 54$

$\boxed{S_6 = 189}$

Sub $S_6 = 189$; $S_7 = -567$ in (1)

$$\overset{88}{189 \times 9} = 1701 \quad 81$$

$$S_9 = -3(-567) - 9(189)$$

$$\overset{22}{567 \times 3} = 1701$$

$$= 1701 - 1701$$

$$\boxed{S_9 = 0}$$

1) If α, β, γ are the roots of the eq $x^3 - 7x + 7 = 0$ find $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$

Solution :-

The Gn eq is $x^3 - 7x + 7 = 0$ ——— (1)

Put $x = \frac{1}{y}$ in (1)

$$\left(\frac{1}{y}\right)^3 - 7\left(\frac{1}{y}\right) + 7 = 0$$

$$\frac{1}{y^3} - \frac{7}{y} + 7 = 0.$$

$\times y^3$ $\quad 1 - 7y^2 + 7y^3 = 0.$

$\div 7$ $\quad y^3 - y^2 + \frac{1}{7} = 0$ — (2).

If α, β, γ are the roots of eq (1).

Then $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are the roots of eq (2).

Find S_4

$$y^3 - y^2 + \frac{1}{7} = 0.$$

Here $r = 4$ $n = 3$

$P_1 = -1$ $\quad P_2 = 0$ $\quad P_3 = 1/7.$

$$S_4 + (-1)S_3 + 0S_2 + \frac{1}{7}S_1 \quad \cdots \quad P_3 S_1 = 0.$$

$$S_4 - S_3 + \frac{1}{7}S_1 = 0.$$

$$S_4 = S_3 - \frac{1}{7}S_1 \quad \text{——— } (3).$$

To find S_1.

Here $r=1$ $n=3$.

$$S_1 = 1S_0 \quad \cdots \quad 1P_1 = 0.$$

$$S_1 + (1)(-1) = 0$$

$$\boxed{S_1 = 1}$$

To find S_3

Here $r=3$ $n=3$.

$$S_3 - 1S_2 + 0S_1 \quad \cdots\cdots \quad 3P_3 = 0$$

$$S_3 - 1S_2 + 3/7 = 0$$

$$S_3 = S_2 - 3/7 \quad \text{——— } (4)$$

To find S_2

Here $r=2$ $n=3$.

$$S_2 - 1S_1 \quad \cdots\cdots \quad 2P_2 = 0.$$

$$S_2 - (1) + 2(0) = 0$$

$$\boxed{S_2 = 1}$$

Sub $S_2 = 1$ in (4)

$$S_3 = 1 - 3/7 \quad \boxed{S_3 = \frac{4}{7}}$$

Sub $S_3 = 4/7$ and $S_1 = 1$

$$S_4 = \frac{4}{7} - \frac{1}{7}(1)$$

$$= \frac{4-1}{7}$$

$$\boxed{S_4 = \frac{3}{7}}$$

(1) Homework

1). If α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$ find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$

2) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$

Prove that $\frac{\alpha^5 + \beta^5 + \gamma^5}{5} = \frac{\alpha^3 + \beta^3 + \gamma^3}{3} \times \frac{\alpha^2 + \beta^2 + \gamma^2}{2}$

Given :-

(ie) :- $\frac{S_5}{5} = \frac{S_3}{3} \times \frac{S_2}{2}$.

$$x^3 + qx + r = 0.$$

$$x^3 + 0x^2 + qx + r = 0.$$

To find S_5 $\quad P_1 = 0 \quad P_2 = q \quad P_3 = r$

Here $r = 5 \quad n = 3$

$r > n$.

$$S_r + P_1 S_{r-1} + P_2 S_{r-2} \cdots P_n S_{r-n} = 0.$$

$$S_5 + 0 S_4 + q S_3 + P_3 S_2 = 0$$

$S_5 + qS_3 + rS_2 = 0$

$S_5 = -qS_3 - rS_2$ ——— ②

To find S_3 $r = 3$ $n = 3$

$S_3 + 0S_2 + qS_1 \ldots\ldots 3P_3 = 0$

$S_3 + qS_1 + 3r = 0$

$S_3 = -qS_1 - 3r$ ——— ③

To find S_2 Here $r = 2$ $n = 3$

$S_2 + 0S_1 + 2P_2 = 0$

$S_2 + 2q = 0$

$\boxed{S_2 = -2q}$

To find S_1 Here $r = 1$ $n = 3$

$S_1 + 0S_0 + 1P_1 = 0$

$\boxed{S_1 = 0}$

Sub $S_1 = 0$ in ③

$S_3 = -q(0) - 3r$

$\boxed{S_3 = -3r}$

Sub $S_3 = -3r$ and $S_2 = -2q$ in ②

$S_5 = -q(-3r) - r(-2q)$

$= 3rq + 2rq$

$\boxed{S_5 = 5rq}$ $\boxed{\frac{S_5}{5} = rq}$ ——— ⑦

$$\frac{S_3}{3} \cdot \frac{S_2}{2} = \frac{-3r}{3} \times \frac{-2q}{2}$$

$$\frac{S_3}{3} \cdot \frac{S_2}{2} = rq \quad \text{———} \quad (8).$$

From (7) & (8).

$$\frac{S_5}{5} = \frac{S_3}{3} \cdot \frac{S_2}{2}.$$

1) If a, b, c, d are the roots of the equation ~~$x^4 + p$~~

$$x^4 + P_2 x^2 + P_3 x + P_4 = 0.$$

Prove that $\frac{a^5+b^5+c^5+d^5}{5} = \frac{a^3+b^3+c^3+d^3}{3} \cdot \frac{a^2+b^2+c^2+d^2}{2}.$

2) Find the sum of 20^{th} power of the roots of equation $x^{10} - x - 2 = 0$

$$x^{10} = x + 2.$$

$$(x^{10})^2 = (x+2)^2$$

$$x^{20} = x^2 + 4x + 4.$$

Here $S_1 = 0 \quad S_2 = 0$

$$\Sigma x^{20} = \Sigma x^2 + \Sigma 4x + 40$$

$$= S_1 + S_2 + 40$$

$$\boxed{\Sigma x^{20} = 40}$$

Hint

$(x^{10})^2 = (x+2)^2$

$S_1 \, \Sigma\alpha = -\frac{2^{nd}}{1^{st}}$

$S_1 = 0$

$S_2 \, \Sigma\alpha\beta = 0$

$S_2 = 0$

METHOD - 1

1) $x^{10} - x - 2 = 0$

$f(x) = x^{10} - x - 2$

$f'(x) = 10x^9 - 1$

$xf'(x) = 10x^{10} - x$

We know that

S_7 = coefficient of $\frac{1}{x^7}$ in the expansion of $\frac{xf'(x)}{f(x)}$

S_{20} = coefficient of $\frac{1}{x^{20}}$ in the expansion of $\frac{10x^{10} - x}{x^{10} - x - 2}$

$$= \frac{\cancel{x^{10}}(10 - 1/x^9)}{\cancel{x^{10}}(1 - 1/x^9 - 2/x^{10})}$$

$$= \frac{(10 - 1/x^9)}{\left(1 - \left(\frac{1}{x^9} + \frac{2}{x^{10}}\right)\right)}$$

$$= \text{''} \quad \left(10 - \frac{1}{x^9}\right)\left(1 - \left(\frac{1}{x^9} + \frac{2}{x^{10}}\right)\right)^{-1}$$

$$= \text{''} \quad \left(10 - \frac{1}{x^9}\right)\left(1 + \left(\frac{1}{x^9} + \frac{2}{x^{10}}\right) + \left(\frac{1}{x^9} + \frac{2}{x^{10}}\right)^2 + \left(\frac{1}{x^9} + \frac{2}{x^{10}}\right)^3\right)$$

$$= \text{''} \quad \left(10 - \frac{1}{x^9}\right)\left(1 + \frac{1}{x^9} + \frac{2}{x^{10}} + \frac{1}{x^8} + \frac{4}{x^{20}}\right)$$

$= \frac{10 \times 4}{x^{20}}$

$= \frac{40}{x^{20}}$

= coefficient of $\frac{1}{x^{20}}$ in the expansion of $\frac{40}{x^{20}}$

$= 40$ //

METHOD - 2

1) $x^{10} - x - 2 = 0$

$x^{10} + 0x^9 + 0x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 - x - 2 = 0$

Here $n = 10$ $r = 20$

$P_1 = 0$ $P_2 = 0$ $P_3 = 0$ $P_4 = 0$ $P_5 = 0$ $P_6 = 0$

$P_7 = 0$ $P_8 = 0$ $P_9 = -1$ $P_{10} = -2$

We know that

$S_r + P_1 S_{r-1} + P_2 S_{r-2} + P_3 S_{r-3} \ldots\ldots P_r S_{n-r} = 0$

Here $r = 20$ $n = 10$ If $r > n$

$S_{20} + 0S_{19} + 0S_{18} + 0S_{17} + 0S_{16} + 0S_{15} + 0S_{14} + 0S_{13}$

$+ 0S_{12} - 1S_{11} - 2S_{10} \ldots\ldots P_{10} S_{10} = 0$

$\therefore S_{20} - 1S_{11} - 2S_{10} = 0$

$S_{20} = S_{11} + 2S_{10}$ ——— (1)

To find S_{11}

Here $r=11$ $n=10$

$$S_{11}+0S_{10}+0S_9+0S_8+0S_7+0S_6+0S_5$$
$$+0S_4+0S_3-1S_2-2S_1\ldots P_{10}S_1=0.$$

$$S_{11}-S_2-2S_1=0.$$

$$S_{11}=S_2+2S_1 \quad\text{—— (2)}.$$

To find S_2

Here $r=2$ $n=10$.

WKT.

$$S_2+0S_1+0S_0\ldots\ldots 2P_2=0$$

$$\boxed{S_2=0}$$

To find S_1

Here $r=1$ $n=10$

$$S_1+0S_0+1P_1=0.$$

$$\boxed{S_1=0}$$

Sub $S_1=0$ and $S_2=0$ in (2)

$$S_{11}=0+2(0)$$

$$\boxed{S_{11}=0}$$

To find S_{10}.

Here $r=10$ $n=10$.

$$S_{10}+0S_9+0S_8+0S_7+0S_6+0S_5+0S_4+0S_3+0S_2\ldots 10P_{10}=0$$

$$S_{10} - 20 + Q(0) = 0$$

$$\boxed{S_{10} = 20}$$

Sub $S_{10} = 20$ and $S_{11} = 0$ in eq (1)

$$S_{20} = S_{11} + 2S_{10}$$

$$= 0 + 2(20)$$

$$\boxed{S_{20} = 40}$$

the 20^{th} power of the roots of the equation is 40

1) If a, b, c, d are the roots of the equation

$$x^4 + P_2 x^2 + P_3 x + P_4 = 0$$

P.T $\frac{a^5+b^5+c^5+d^5}{5} = \frac{a^3+b^3+c^3+d^3}{3} \times \frac{a^2+b^2+c^2+d^2}{2}$.

Solution :-

The Gn equation $x^4+0x^3+P_2x^2+P_3x+P_4=0$

$P_1 = 0$; $P_2 = P_2$; $P_3 = P_3$; $P_4 = P_4$ ..

To find S_5

Here $r=5$ $n=4$.

WKT. If $r>n$.

$S_5 + 0S_4 + P_2 S_3 + P_3 S_2 + P_4 S_1 \ldots\ldots P_4 S_1 = 0$.

$S_5 + P_2 S_3 + P_3 S_2 + P_4 S_1 = 0$.

$S_5 = -P_2 S_3 - P_3 S_2 - P_4 S_1$ ——— (1)

To find S_3

Here $r=3$; $n=4$

$S_3 + 0S_2 + P_2 S_1 \ldots\ldots + 3P_3 = 0$ If $r \le n$

$S_3 + P_2 S_1 + 3P_3 = 0$

$S_3 = -P_2 S_1 - 3P_3$ ——— (2)

To find S_2

Here $r=2$ $n=4$

$S_2 + 0S_1 \ldots\ldots 2P_2 = 0$

$S_2 + 2P_2 = 0$

$$S_2 = -2P_2 \quad \text{———} \quad (3)$$

To find S_1

Here $r=1 \quad n=4$

$$S_1 + 0.S_0 + \dots 1P_1 = 0$$

$$\boxed{S_1 = 0}$$

Sub $S_1 = 0$ in (2)

$$S_3 = 0 - 3P_3$$

$$\boxed{S_3 = -3P_3} \quad \text{———} \quad (4)$$

Sub $S_3 = -3P_3$, $S_2 = -2P_2$ and $S_1 = 0$

$$S_5 = -P_2(-3P_3) - P_3(-2P_2) + 0$$

$$= 3P_2P_3 + 2P_2P_3$$

$$= 5P_2P_3$$

$$S_5 = 5P_2P_3 \quad \text{———} \quad (5)$$

$$\frac{S_5}{5} = P_2P_3 \quad \text{———} \quad (6)$$

We know that

$$S_3 = -3P_3 \qquad S_2 = -2P_2$$

$$\frac{S_3}{3} = -P_3 \qquad \frac{S_2}{2} = -P_2$$

$$\frac{S_3}{3} \cdot \frac{S_2}{2} = (-P_2)(-P_3) = P_2P_3 \quad \text{———} \quad (7)$$

From (6) and (7) $\quad \frac{S_5}{5} = \frac{S_3}{3} \times \frac{S_2}{2} = P_2P_3$

Hence it is proved.