17UMAE03

DISCRETE MATHEMATICS

III B.SC MATHEMATICS

V SEMESTER

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DISCRETE MATHEMATICS

UNIT - I

Mathematical logic - Statemends and Notations- Connectives - Negation - conjuction - Disjunction - Statement formulas and Truth table - Conditional and Bi-conditional - Well formed formulas - Tautologies. (sections 1.1, 2.1, to 1.2.4, 1.2.6 to 1.2.8)

UNIT - II

Normal forms - Disjunctive Normal forms - conjunctive Normal forms - principal Disjunctive Normal forms - principal conjunctive Normal forms - ordering and uniqueness of Normal forms the theory of inference for the statement calculus - validity using truth tables - Rules of inference.
(sections 1.3.1 to 1.3.5, 1.4.1 to 1.4.2)

UNIT - III

The predicate calculus - predicates - the statements function, variables and quantifiers - predicate formulas - Free and bound variables - the universe of discourse - Inference theory of the predicate calculus - valid formulas and Equivalence - some valid formulas over finite universes - special valid formulas involving quantifiers - Theory of inference for the predicate calculus. (section 1.6.1 to 1.6.4)

UNIT - IV

Relations and ordering - Relations - properties of binary relations in a set - partial ordering - partially ordered set : Representation and associated terminology Functions - Definition and Introduction - composition of functions - Inverse functions - Natural numbers - Peano axioms - Mathematical Induction.

(Sections 2.3.1, 2.3.2, 2.3.8, 2.3.9, 2.4.1, 2.4.3, 2.5.1).

UNIT - V

Lattices a partially ordered sets: Definition and Examples - some properties of Lattices - Boolean Algebra: Definition and Examples - sub algebra, Direct Product and homomorphism - Boolean functions - Boolean forms and free Boolean algebra - values of Boolean expression and Boolean functions.

(Sections: 4.1.1, 4.1.2, 4.2.1, 4.2.2, 4.3.1, 4.3.2)

Text Book:

1. J.P. Trembly, R. Manohar, Discrete Mathematical structure with applications to computer science, Tata MC Graw Hill 2001.

Reference Book:

1. Dr. M.K. Sen and Dr. B.C. charaborthy, Introduction to Discrete Mathematics, Arunabhd sen Books and Allied . Pvt. Ltd., 8/1 chintamoni Daslane, kolkata - 700009 Reprinted in 2016.

DISCRETE MATHEMATICS

UNIT - I

Mathematical logic - statements and Notations - connectives - Negation - conjuction - Disjunction - statement formulas and Truth table - conditional and Bi-conditional - Well formed formulas - Tautologies.
(Sections 1.1, 2.1, to 1.2.4, 1.2.6 to 1.2.8)

UNIT - I

TRUTH VALUES:

There are two truth values. They are TRUE and FALSE. They are also denoted by T and F (or) 1 and 0 respy.

Statement:

A statement is a declarative sentence that can be assigned any one of the two truth values. Statements are always denoted by the capital letters A to Z.

Ex: P : India is a country.
Q : 5 is a rational number.

Atomic statement:

The statements which do not contain any connectives is called a atomic (or) primary statements.

Ex: C: France is a country.

Compound statement:

The statements formed from the atomic statements through the use of sentential connectives are called compound (or) composite (or) molecular statements.

Those statements contains one (or) more primary statements and some connectives are called molecular statements.

Ex: 122 is a prime number and 2015 is not a leap year.

1. Connectives :

1. Negation ($\neg$) unary operator :

The Negation of a statement is formed by introducing the word "**not**" at a proper place in the statement (or) by prefixing the following phrases.

(i) "It is False that"
(ii) "It is not true that"
(iii) "It is not the case that"

Ex : P : Ramu went to school yesterday.
$\neg$P : "It is not true that" Ramu went to school yesterday.

Truth table for Negation:

P	$\neg$P
T	F
F	T

2. conjunction ($\wedge$) Binary operator:

The conjunction of true statements $P \wedge Q$ which is read as "p and Q".

Ex:
Let P: New Delhi is a capital of India.
Q: $15+15=15$
Then $P \wedge Q$: New Delhi is the capital of India and $15+15=15$.

Truth table for conjunction :

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction ($\vee$) Binary operator:

The Disjunction of two statements P and Q is the statement $P \vee Q$ which is read as "P (or) Q".

Ex:

Let P: Sikkim is a state of India.

and Q: 16 is divisible by 3.

Then $P \vee Q$: Sikkim is a state of India or 16 is divisible by 3.

Truth table for disjunction ($\vee$):

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

4. Conditional statement ($\to$):

If P and Q are any two statements then the statement $P \to Q$ which is read as "if P then Q" is called a conditional statement.

The statement P is called anticedent and the statement Q is called consequent.

The represent any one of the following expressions to $P \to Q$

(i) Q is neccessary for P

(ii) P is sufficient for Q

(iii) Q if P

(iv) P only if Q

(v) P implies Q

Truth table for conditional:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex: P: Today is Monday.

Q: 28 is a perfect square.

$P \to Q$: If today is Monday then 28 is a perfect square.

Bi-Conditional statement ($\rightleftarrows$)

If P and Q are any two statement then the $P \rightleftarrows Q$ two statements which is read as "P if and only if Q".

Truth table for Biconditional ($\rightleftarrows$)

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

Symbolise the following statements:

1. If Either Jerry takes calculus or ken takes algebra then Lara will take dynamics.

soln:

Let P: Jerry takes calculus.

Q: Ken takes algebra

R: Lara will take dynamics.

$\therefore$ The given statement can be symbolised as $(P \vee Q) \to R$.

2. The crops will be destroyed if there is a flood.

soln:

The given statement can be return as If there is a flood. then the crops will be destroyed.

Let as denote

A: There is a flood.

B: The crops will be destroyed.

$\therefore$ The given statement can be symbolised as $A \to B$.

3. Using the statements

R: Mark is rich

H: Mark is happy

while the following statements in the symbolic form

i) Mark is poor but happy.

ii) Mark is rich or unhappy.

iii) Mark is neither rich nor happy.

iv) Mark is poor or he is both rich and unhappy.

Soln:

i) Mark is poor but happy : $\neg R \wedge H$

ii) Mark is rich or unhappy : $R \vee \neg H$

iii) Mark is neither rich nor happy : $\neg R \wedge \neg H$

iv) Mark is poor or he is both rich and unhappy:

$\neg R \vee (R \wedge \neg H)$

4. Jack and Jill went up the hill.

soln:

A: Jack went up the hill

B: Jill went up the hill

∴ The given statement can be symbolised as $A \cap B$.

Statement formula:

A statement formula is an Expression which is a string consisting of variables [capital letters with (or) without subscribes] parantheses and connective symbols.

Ex: $(P \rightleftarrows Q) \rightarrow (\neg R \rightarrow (P \vee \neg Q))$

Well Formed Formula [WFF]:

A well formed formula can be generated by the following rules.

1. A statement variable standing alone is a WFF.

2. If A is a WFF, then $\neg A$ is also a WFF.

3. If A and B are WFFs, then $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \rightleftarrows B)$ are WFFs.

4. A string of symbols containing the statement variables, connectives and parantheses is a WFF.

"if and only if" (iff) it can be obtained by finitely many applications of the rules (1, 2 and 3).

Examples for WFFs:

$\neg(P \wedge Q)$, $\neg(P \vee Q)$, $(P \rightarrow (P \vee Q))$, $(P \rightarrow (P \rightarrow R))$, $[((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightleftarrows (P \rightarrow R)]$

The following are not well formed formulas.

1. $\neg P \wedge Q$

Either this would be $(\neg P \wedge Q)$ (or) $(\neg(P \wedge Q))$, then it is WFF.

2. $P \rightarrow$

$P \rightarrow$ is not allowed.

3. $(P \wedge Q) \rightarrow Q$

Paranthoses are mismatching.

4. $(P \rightarrow Q) \rightarrow (\wedge R)$

Conjunction R is not allowed.

Tautology : (T)

A statement formula which is true regardless of the truth values of the statements which replace the variables in it is called a universally valid formula (or) a Tautology (or) a logical truth.

Ex: $P \vee \neg P$ is a tautology where, P is any statement formula.

Contradiction : (F)

A statement formula which is false regardless of the truth values of the statements which replace the variables in it is called a contradiction.

Ex: $P \wedge \neg P$ is a contradiction where, P is any statement formula.

Construct the truth table for the statement formula:

1. $S: \neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$\neg P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$	$S: \neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	T	T	F
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F	F	F
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	F	F	F

$\therefore$ The given statement formula is a contradiction.

2. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$.

P	Q	R	$(Q \rightarrow R)$	$(P \rightarrow (Q \rightarrow R))$	$(P \rightarrow Q)$	$(P \rightarrow R)$	$((P \rightarrow Q) \rightarrow (P \rightarrow R))$	$(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	F	T
T	F	T	T	F	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

$\therefore$ The given statement formula is a Tautology.

3. $(P \to (Q \to S)) \wedge ((\neg R \vee P) \wedge Q)$

P	Q	R	S	$Q \to S$	$(P \to (Q \to S))$	$\neg R$	$(\neg R \vee P)$	$((\neg R \vee P) \wedge Q)$	$(P \to (Q \to S)) \wedge ((\neg R \vee P) \wedge Q)$
T	T	T	T	T	T	F	T	T	T
T	T	T	F	F	F	F	T	T	F
T	T	F	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T	T	F
T	F	T	T	T	T	F	T	F	F
T	F	T	F	T	T	F	T	F	F
T	F	F	T	T	T	T	T	F	F
T	F	F	F	T	T	T	T	F	F
F	T	T	T	T	T	F	F	F	F
F	T	T	F	F	T	F	F	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	F	T	T	T	T	T
F	F	T	T	T	T	F	F	F	F
F	F	T	F	T	T	F	F	F	F
F	F	F	T	T	T	T	T	F	F
F	F	F	F	T	T	T	T	F	F

∴ The given statement formula is neither a tautology nor a contradiction.

Equivalence of propositions:

The two compound propositions $A[P_1, P_2, P_3 \dots P_n]$ and $B[P_1, P_2, P_3 \dots P_n]$ are said to be logically equivalent (or) simply equivalent if the truth value of A is equal to the truth value of B for everyone of the 2^n possible sets of truth values assigned to $P_1, P_2, P_3 \dots P_n$.

Note:

1. If the two compound propositions A and B are equivalent then it is denoted by $A \Leftrightarrow B$.
2. $A \Leftrightarrow B$ if and only if (iff) $A \rightleftarrows B$ is a tautology.

1. Write the duals of the following.

a) $(P \wedge Q) \vee R$ – $(P \vee Q) \wedge R$

b) $(P \vee Q) \wedge T$ – $(P \wedge Q) \vee F$

c) $(P \to Q) \vee F$ – $(P \to Q) \wedge T$

d) $\neg(P \vee Q) \wedge (P \wedge (Q \vee \neg S))$ – $\neg(P \wedge Q) \vee (P \vee (Q \wedge \neg S))$

Equivalent Formulas:

S.No	Name of the Law	Primal form	Dual form
1.	Idempotent law	$P \vee P \Leftrightarrow P$	$P \wedge P \Leftrightarrow P$
2.	Identity law	$P \vee F \Leftrightarrow P$	$P \wedge T \Leftrightarrow P$
3.	Dominant law	$P \vee T \Leftrightarrow T$	$P \wedge F \Leftrightarrow F$
4.	Complementry law	$P \vee \neg P \Leftrightarrow T$	$P \wedge \neg P \Leftrightarrow F$
5.	commutative law	$P \vee Q \Leftrightarrow Q \vee P$	$P \wedge Q \Leftrightarrow Q \wedge P$
6.	Associative law	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
7.	Distributive law	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
8.	Absorbtion law	$P \vee (P \wedge Q) \Leftrightarrow P$	$P \wedge (P \vee Q) \Leftrightarrow P$
9.	De Morgan's law	$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

where $\mathbb{T}$ denotes tautology.
where $\mathbb{F}$ denotes contradiction.

Duality law:

The compound propositions A and A' are said to be Duals of each other if either one can be obtained from the other by replacing

$\wedge$ by $\vee$,
$\vee$ by $\wedge$,
$\mathbb{T}$ by $\mathbb{F}$,
$\mathbb{F}$ by $\mathbb{T}$.

Prove that $P \to Q \Leftrightarrow \neg P \vee Q$

P	Q	$P \to Q$	$\neg P$	$\neg P \vee Q$	$(P \to Q) \Leftrightarrow (\neg P \vee Q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$\therefore$ Hence $P \to Q \Leftrightarrow \neg P \vee Q$

Results:

1. $P \rightleftarrows Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$
2. $P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$