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OPERATIONS RESEARCH

B.Sc MATHEMATICS

V - SEMESTER

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Operations Research

UNIT - I

Introduction - Definition of OR - Scope, Phrases and Limitations of OR - Linear Programming problem - Graphical Method - Definitions of bounded, unbounded and optimal solutions - procedure of solving LPP by graphical method - problems - simplex technique - Definitions of Basic, non-basic variables - basic solutions - slack variable, surplus variables and optimal solution, simplex Procedure of solving LPP - problems.

1. Definition of O.R:-

O.R is the application of scientific methods, techniques and tools to problems Involving the operations of a system so as to procide there in control of the system with optimum solutions to the problem.

2. Phases of O.R:-

i). Formulating the problem
ii). constructing the Model
iii). Deriving the solution
iv). updating the Model
v). controlling the solution

3. Limitating of O.R:

i). optimum use of production factors:

Linear programming techniques indicate how a manager can most effectively employ his production factors by more efficiently selecting and distributing these elements.

ii). Improved quality of decision:

The computation table gives a clear picture of the happenings within the basic restrictions and the possibility of compound behaviour of the elements invalved in the production pattern problem. The effect on the production pattern will be clearly indicated in the table.

e.g: simplex Table.

iii). Preparing of future managers:

These methods substitude a means for improving the knowledge and skill of young managers.

4. Modification of Mathematical solution:

O.R. presents a possible practical saluction when one exists, but it is always a responsibility of the manager to accept or modify the saluction before its use. The effect of these modifications may be evaluated from the computational steps and tables.

5. Alternative solution:

O.R. techniques will suggest all the alteractive solutions available for the same profit. So that the management may decide on the basis of its strategies.

Unit -I

Linear programming problem (LPP)

General form of LPP:-

$$\text{Max } Z \text{ (or) } \text{Min } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

Subject to the constraints,

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq \text{ or } \geq \text{ or } = b_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n \leq \text{ or } \geq \text{ or } = b_2$$

$$\vdots \quad \vdots \quad + \quad + \quad .$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \leq \text{ or } \geq \text{ or } = b_n$$

and $x_1, x_2, x_3, \dots, x_n \geq 0.$

Optimum solutions:

Any feasible solns which optimize the objective functions of LPP is is called optimum solutions.

Procedure for Graphical Method:-

Step 1:

Draw x_1 and x_2 axis on a graph sheet.

Step 2:

Draw a line and identify the region connected with it corresponding to each constraints.

Step 3:

Identify the solutions space which is the region that is common to all the constraints including the non-negativity restrictions.

Step 4:

Find the value of z at each vertex of the solns space.

Step 5:

Identify the optimum solutions.

Graphical Method:-

1. Solve the LPP using graphical method Max $z = 2x_1 + 4x_2$. subject to, $x_1 + 2x_2 \leq 5$ $x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$.

Sol:

Convert the given inequalities into equality.

$x_1 + 2x_2 = 5 \rightarrow (1)$

$x_1 + x_2 = 4 \rightarrow (2)$

Put $x_1 = 0$ in eqn (1)

$2x_2 = 5$

$x_2 = 5/2 \Rightarrow 2.5$

The points $(0, 2.5)$

Put $x_2 = 0$ in (1)

$x_1 + 0 = 5$

$x_1 = 5$

The points $(5, 0)$

Put $x_1 = 0$ in (2)

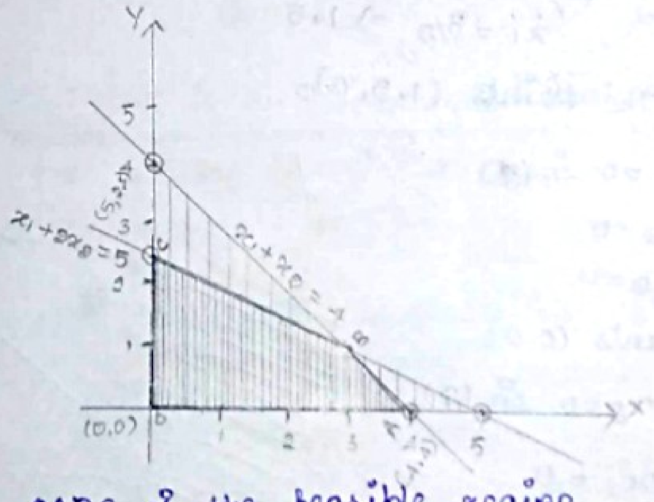
$x_2 = 4$

The points $(0, 4)$

Put $x_2 = 0$ in (2)

$x_1 = 4$

The points $(4, 0)$



OABC is the feasible region

To find B:-

$$\begin{aligned} \cancel{x_1} + 2x_2 &= 5 \\ \cancel{x_1} + x_2 &= 4 \\ (-) \quad (-) \quad & (-) \\ \hline x_2 &= 1 \end{aligned}$$

put $x_2 = 1$, in (2)

$x_1 + 1 = 4$

$x_1 = 4 - 1$

$x_1 = 3$

The point $B(3, 1)$

Boundary points $\quad$ Max $Z = 2x_1 + 4x_2$

$O(0,0)$ $\quad Z = 0$

$A(4,0)$ $\quad Z = 8$

$B(3,1)$ $\quad Z = 10$

$C(0,2.5)$ $\quad Z = 10$

The optimum solution is

Max $Z = 10$ $\quad$ Max $Z = 10$

$x_1 = 3;\ x_2 = 1$ $\quad$ $x_1 = 0;\ x_2 = 2.5$

2. Solve the LPP by graphical method. Max $Z = 6x_1 + x_2$ subject to constraints $2x_1 + x_2 \geq 3$, $x_1 - x_2 \geq 0$ and $x_1, x_2 \geq 0$.

Sol:

Convert the given inequalities into equality

$$2x_1 + x_2 = 3 \rightarrow (1)$$

$$x_1 - x_2 = 0 \rightarrow (2)$$

put $x_1 = 0$ in eqn (1)

$x_2 = 3$

The points $(0,3)$

put $x_2 = 0$ in eqn (1)

$2x_1 = 3$

$x_1 = 3/2 \Rightarrow 1.5$

The points $(1.5, 0)$

Put $x_1 = 0$ in (2)

$-x_2 = 0$

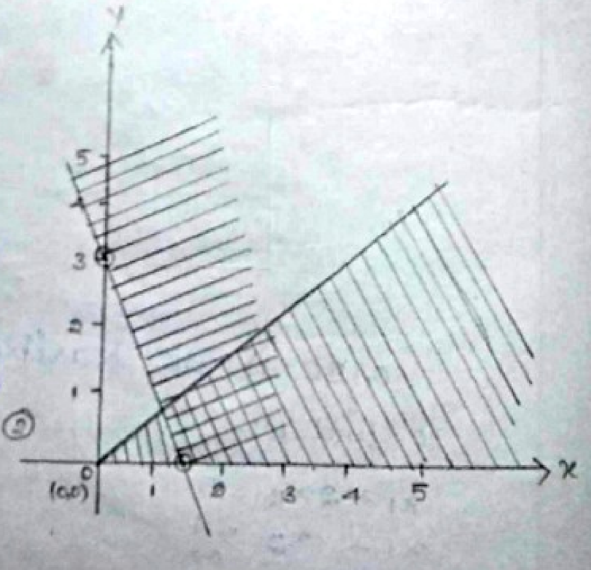
$x_2 = 0$

The points $(0,0)$

put $x_2 = 0$ in (2)

$x_1 = 0$

The points $(0,0)$



$\therefore$ There is an unbounded solutions.

3. Solve the LPP by graphical method. $\text{Max } Z = x_1 + x_2$
Subject to, $x_1 + x_2 \le 1$, $-3x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$.

sol:

convert the given inequalities into equality.

$$x_1 + x_2 = 1 \rightarrow (1)$$

$$-3x_1 + x_2 = 3 \rightarrow (2)$$

put $x_1 = 0$ in eqn (1)

$x_2 = 1$

The points $(0,1)$

put $x_2 = 0$ in (1),

$x_1 = 1$

The points $(1,0)$

put $x_1 = 0$ in (2),

$x_2 = 3$

The points $(0,3)$

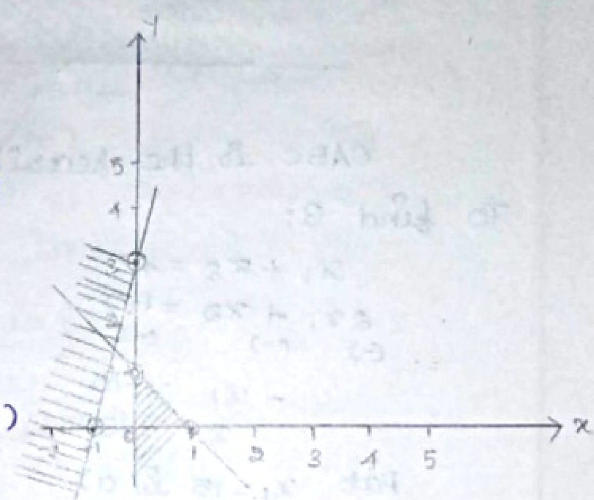
put $x_2 = 0$ in (2)

$-3x_1 = 3$

$x_1 = -3/3$

$x_1 = -1$

The points $(-1,0)$



∴ There is an unbounded solutions.

4. Solve the LPP by graphical method $\text{MaxZ} = 3x_1 + 4x_2$
Subject to $x_1 + x_2 \le 45$, $2x_1 + x_2 \le 60$ and $x_1, x_2 \ge 0$.

sol:

convert the given inequalities into equality.

$$x_1 + x_2 = 45 \rightarrow (1)$$

$$2x_1 + x_2 = 60 \rightarrow (2)$$

put $x_1 = 0$ in (1),

$x_2 = 45$

The points $(0,45)$

put $x_2 = 0$ in (1)

$x_1 = 45$

The points $(45,0)$

put $x_1 = 0$ in (2)

$x_2 = 60$

The points $(0, 60)$

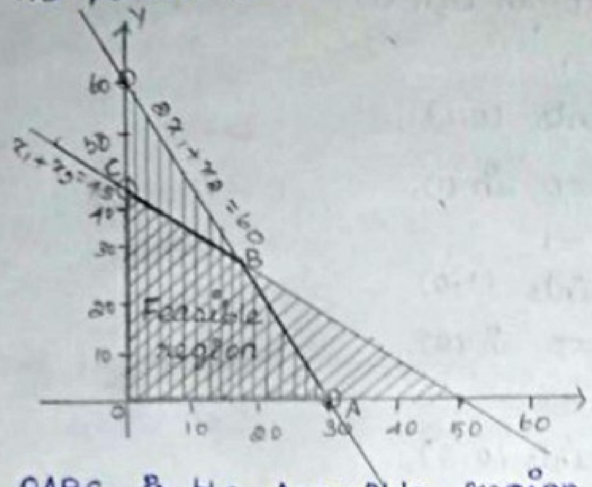
Put $x_2 = 0$ in (2)

$2x_1 = 60$

$x_1 = 60/2$

$x_1 = 30$

The points $(30, 0)$



OABC is the feasible region

To find B:

$x_1 + x_2 = 45$

$2x_1 + x_2 = 60$

$(-) \quad (-) \quad (-)$

$-x_1 = -15$

$x_1 = 15$

Put $x_1 = 15$ in (1)

$15 + x_2 = 45$

$x_2 = 45 - 15$

$x_2 = 30$

The point $B(15, 30)$

Boundary points	$Max\,Z = 3x_1 + 4x_2$
$O(0, 0)$	$Z = 0$
$A(30, 0)$	$Z = 90$
$B(15, 30)$	$Z = 165$
$C(0, 45)$	$Z = 180$

∴ The optimum solution is

$Max\,Z = 180$

$x_1 = 0; \; x_2 = 45.$

5. Solve : $Max\,Z = 4x_1 + 3x_2$ subject to, $2x_1 + x_2 \leq 1000$,

$x_1+x_2 \le 800$, $x_1 \le 400$, $x_2 \le 700$, and $x_1, x_2 \ge 0$.

soln:

Convert the given inequlities into equality.

$$2x_1+x_2 = 1000 \rightarrow (1)$$
$$x_1+x_2 = 800 \rightarrow (2)$$
$$x_1 = 400 \rightarrow (3)$$
$$x_2 = 700 \rightarrow (4)$$

Put $x_1=0$ in eqn (1),

$x_2 = 1000$

The points $(0,1000)$

put $x_2=0$ in (1)

$2x_1 = 1000$

$x_1 = 1000/2$

$x_1 = 500$

The points $(500,0)$

put $x_1=0$ in (2),

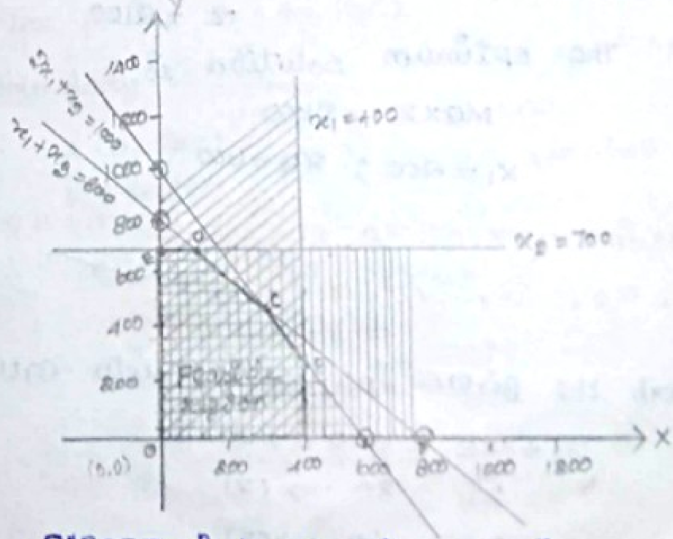
$x_2 = 800$

The points $(0,800)$

put $x_2=0$ in (2)

$x_1 = 800$

The points $(800,0)$



OABCDE is the feasible region

To find B:	To find D:
$x_1 = 400$	$x_1+x_2 = 800$
$2x_1+x_2 = 1000$	$x_2 = 700$
$2(400)+x_2 = 1000$	$x_1+700 = 800$

$800 + x_2 = 1000$

$x_2 = 1000 - 800$

$x_2 = 200$

The points B (400, 200)

$x_1 = 800 - 700$

$x_1 = 100$

The points D (100, 700)

To find C:-

$2x_1 + x_2 = 1000$

$x_1 + x_2 = 800$

(-) (-) (-)

$x_1 = 200$

$200 + x_2 = 800$

$x_2 = 800 - 200$

$x_2 = 600$

The points C (200, 600)

Boundary points $\qquad$ Max $Z = 4x_1 + 3x_2$

O (0, 0) $\qquad Z = 0$

A (400, 0) $\qquad Z = 1600$

B (400, 200) $\qquad Z = 1600 + 600 = 2200$

C (200, 600) $\qquad Z = 800 + 1800 = 2600$

D (100, 700) $\qquad Z = 400 + 2100 = 2500$

E (0, 700) $\qquad Z = 2100$

The optimum solution is

Max $Z = 2600$

$x_1 = 200$; $x_2 = 600$

6. Solve Min $Z = 3x_1 + 5x_2$ subject to $x_1 + x_2 \geq 200$, $x_1 \leq 80$, $x_2 \geq 60$, and $x_1, x_2 \geq 0$.

Sol:

Convert the given inequalities into equality.

$x_1 + x_2 = 200 \rightarrow (1)$

$x_1 = 80 \rightarrow (2)$

$x_2 = 60 \rightarrow (3)$

Put $x_1 = 0$ in (1)

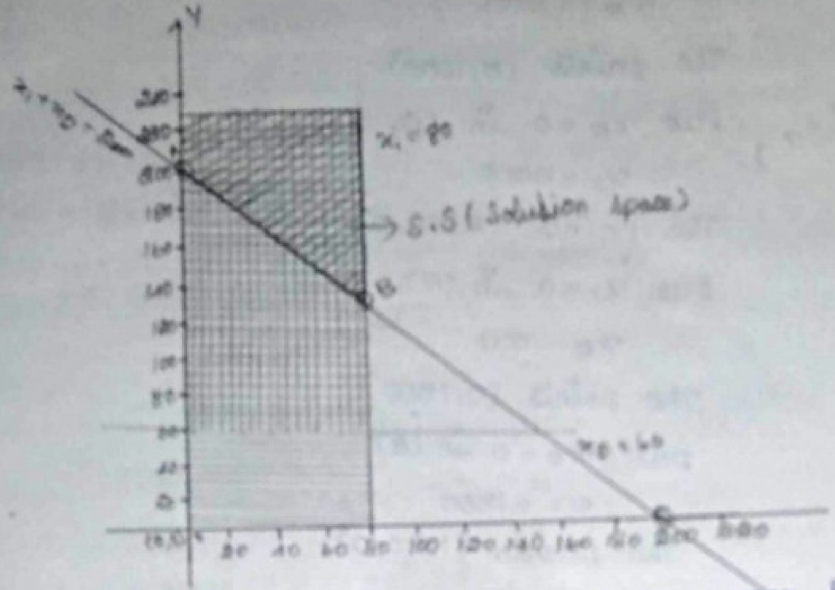
$x_2 = 200$

The points (0, 200)

put $x_2 = 0$ in (1),

$x_1 = 200$

The points $(200, 0)$



It is unbounded there is no upper boundary in this region. The vertices of the common region A and B its co-ordinates are A (0, 200)

To find B:

$x_1 = 80$

$x_1 + x_2 = 200$

$80 + x_2 = 200$

$x_2 = 200 - 80$

$x_2 = 120$

The points B (80, 120)

Boundary points	Min $Z = 3x_1 + 5x_2$
A (0, 200)	$Z = 1000$
B (80, 120)	$Z = 240 + 600$
	$Z = 840$

The optimum solution is

Min $Z = 840$

$x_1 = 80$; $x_2 = 120$.

7 Solve : Max $Z = x_1 + x_2$ subject to $x_1 + 2x_2 \le 2000$, $x_1 + x_2 \le 1500$, $x_2 \le 600$ and $x_1, x_2 \ge 0$.

soln:

Convert the given inequalities into equality.

$x_1 + 2x_2 = 2000 \rightarrow (1)$

$x_1 + x_2 = 1500 \rightarrow (2)$

$x_2 = 600 \rightarrow (3)$

Put $x_1 = 0$ in (1),

$2x_2 = 2000$

$x_2 = 2000/2$

$x_2 = 1000$

The points $(0, 1000)$

Put $x_2 = 0$ in (1),

$x_1 = 2000$

The points $(2000, 0)$

$x_1 + x_2 = 1500$

$0 + x_2 = 1500$

$x_2 = 1500$

Put $x_1 = 0$ in (2),

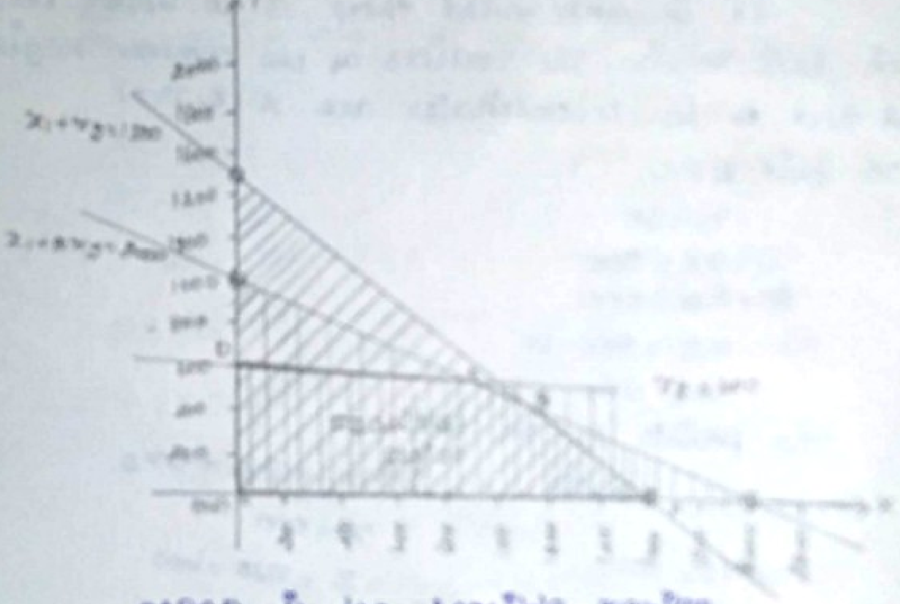
$x_2 = 1500$

The points $(0, 1500)$

Put $x_2 = 0$ in (2)

$x_1 = 1500$

The points $(1500, 0)$



OABCD is the feasible region.

To find B :

$x_1 + 2x_2 = 2000$

$x_1 + x_2 = 1500$

(-) (-) (-)

$x_2 = 500$

$x_1 + 500 = 1500$

$x_1 = 1500 - 500$

$x_1 = 1000$

∴ The points B(1000, 500)

To find C :

$x_2 = 600$

$x_1 + 2x_2 = 2000$

$x_1 + 2(600) = 2000$

$x_1 = 2000 - 1200$

$x_1 = 800$

∴ The points C (800, 600)

Boundary points $\qquad$ $Max z = x_1 + x_2$

$O(0,0)$ $\qquad$ $z = 0$

$A(1500,0)$ $\qquad$ $z = 1500$

$B(1000,500)$ $\qquad$ $z = 1000 + 500 = 1500$

$C(800,600)$ $\qquad$ $z = 800 + 600 = 1400$

$D(0,600)$ $\qquad$ $z = 600$

$\therefore$ The optimum solution is

$$Max z = 1500$$
$$x_1 = 1500 ; \; x_2 = 0$$
$$Max z = 1500$$
$$x_1 = 1000 ; \; x_2 = 500$$

8. Solve : $Max z = 3x_1 + 2x_2$ subject to, $-2x_1 + x_2 \leq 1$, $x_1 \leq 2$, $x_1 + x_2 \leq 3$ and $x_1, x_2 \geq 0$.

Sol: convert the given inequalities into equality.

$$-2x_1 + x_2 = 1 \rightarrow (1)$$
$$x_1 = 2 \rightarrow (2)$$
$$x_1 + x_2 = 3 \rightarrow (3)$$

Put $x_1 = 0$ in (1)

$x_2 = 1$

The points $(0,1)$

Put $x_2 = 0$ in (1),

$-2x_1 = 1$

$x_1 = -1/2$

$= -0.5$

The points $(-0.5, 0)$

Put $x_1 = 0$ in (3)

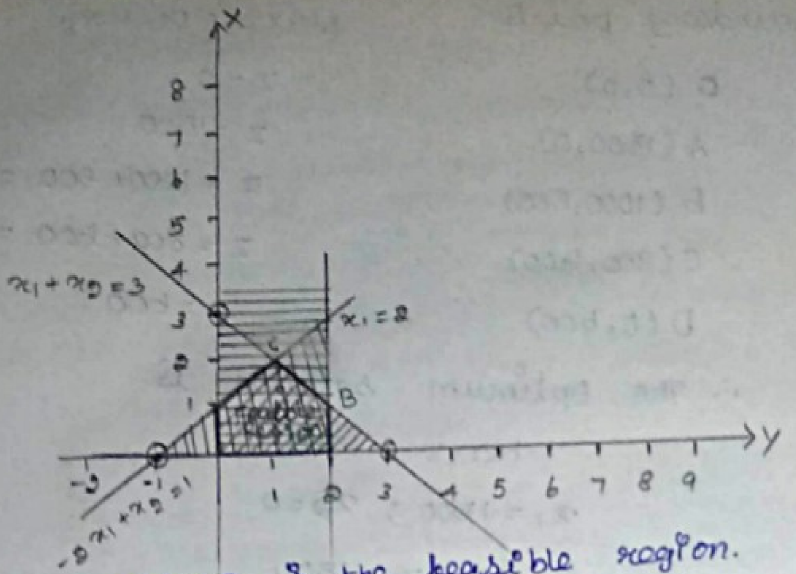
$x_2 = 3$

The points $(0,3)$

Put $x_2 = 0$ in (3)

$x_1 = 3$

The points $(3,0)$



OABCD is the feasible region.

To find B:

$x_1=2$

$x_1+x_2=3$

$2+x_2=3$

$x_2=3-2$

$x_2=1$

The points B (2,1)

To find C:

$x_1+x_2=3$

$-2x_1+x_2=1$

(+) (-) (-)

$3x_1=2$

$x_1=2/3$

$x_1=0.7$

(3) $\Rightarrow 0.7+x_2=3$

$x_2=3-0.7$

$x_2=2.3$

The points C (0.7, 2.3)

Boundary points

O (0,0)

A (2,0)

B (2,1)

C (0.7, 2.3)

D (0,1)

Max $z = 3x_1+2x_2$

$z=0$

$z=6$

$z=6+2=8$

$z=2.1+4.6=6.7$

$z=2$

$\therefore$ The optimum solution is

Max $z=8$

$x_1=2$; $x_2=1$.

Def:

Solutions:

A set of values $x_1, x_2, \dots$ which satisfys the constraints of the LPP is called its solutions.

Feasible Solutions:

Any solutions to a LPP which satisfies the non-negative restrictions of the LPP is called feasible solutions.

Slack variable:

If the constraints of a general LPP be,

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i = 1, 2, 3, \dots K) \rightarrow (1)$$

Then the non-negative variables and which are introduce to convert the inequalities eqn (1) to the equalities

$$\sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \quad (i = 1, 2, 3, \dots K) \text{ are}$$

called slack variable.

Canonical form:

The general linear programming problem can always be express in the following form

$$\text{Maximize} = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

s.t.c

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n \leq b_2$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \leq b_m$$

and the non-negativity restriction

$$x_1, x_2, \dots, x_n \geq 0.$$

Standard form:

The general linear programming problem in the form

$$\text{Maximize } z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

s.t.c

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m$$

and the non-negativity restriction $x_1, x_2, \dots x_n$.

Optimum solutions:

Any feasible solutions which optimise the objective functions of LPP is called optimum solutions.

Procedure for Graphical Method:

Step 1:

Draw x_1 and x_2 axis on a graph sheet.

Step 2:

Draw a line and identify the region connected with its corresponding to each constraints.

Step 3:

Identify the solns space which is the region that is common to all the constraints including the non-negativity restrictions.

Step 4:

Find the value of z at each vertex of the solns space.

Step 5:

Identify the optimum solns.

Surplus variable.

Let the constraints of a general LPP be

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad i = k+1, k+2, \dots, l$$

Then the non-negative variables y_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} x_j - x_{mi} = b_i \quad i = k+1, k+2, \dots l$$

Basic solution:

Given a system of m simultaneous linear equation in n unknowns $(m < n)$

$$Ax = b, \; x^T \in R^n$$

where A is an $m \times n$ matrix of rank m

Let A is any $m \times n$ submatrix formed by m linearly independent columns of A. Then a solution obtained by setting n-m variables not associated with the column of B, equal to zero and solving the resulting system is called basic solution to the given system of equation.

Degenerate solution:

A basic solution to the system is called degenerate if one or more of the basic variable vanish.

Simplex method:

Q 1. use simplex method to solve following LPP

$Max\ z = 7x_1 + 5x_2$ subject to, $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12$ and $x_1, x_2 \geq 0$. $\longrightarrow$ $(-x_1 - x_2 \geq -2)$ $(x_1 + x_2 \leq 2)$

soln: canonical form:

$$Max\ z = 7x_1 + 5x_2 \text{ subject to,}$$

$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$\text{and } x_1, x_2 \geq 0$$

Standard form:

$$Max\ z = 7x_1 + 5x_2 + 0S_1 + 0S_2$$

$$\text{S.T.C } x_1 + 2x_2 + S_1 = 6$$
$$4x_1 + 3x_2 + S_2 = 12$$
$$\text{and } x_1, x_2, S_1, S_2 \geq 0.$$

Matrix form:

$$\begin{array}{cccc} x_1 & x_2 & S_1 & S_2 \end{array}$$
$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$

Starting table:

			c_j 7	5	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	Ratio
z_j 0	S_1	6	1	2	1	0	$\frac{6}{1} = 6$
z_j 0	S_2	12	4	3	0	1	$\frac{12}{4} = 3$
$z_j - c_j$			-7	-5	0	0	

S_2 leaves the basis

x_1 enter the basis

old. eqn : 12 4 3 0 1

new eqn : $\frac{12}{4}=3$ $\frac{4}{4}=1$ $\frac{3}{4}=\frac{3}{4}$ 0 $\frac{1}{4}$

change:

$$\begin{array}{rrrrrr} & 6 & 1 & 2 & 1 & 0 \\ (-) & 3 & 1 & 3/4 & 0 & 1/4 \\ \hline & 3 & 0 & 5/4 & 1 & -1/4 \end{array}$$

I^{st} - Iteration :-

		c_j column	7	5	0	0	
	C_B	Y_B	X_B	x_1	x_2	S_1	S_2
z_j	0	S_1	3	0	5/4	1	-1/4	row
z_j	7	x_1	3	1	3/4	0	1/4	row
	$z_j - c_j$		21	0	1/4	0	7/4

Here all $z_j - c_j \geq 0$

$\therefore$ The optimum solution is

$\text{Max } z = 21, \; x_1 = 3$ and $x_2 = 0$.

2. use simplex method to solve the following LPP.

$\text{Max} z = 10x_1 + 20x_2$ subject to, $3x_1 + 5x_2 \leq 90$, $6x_1 + 3x_2 \leq 72$ and $x_1, x_2 \geq 0$.

sol:

canonical form:

$\text{Max} z = 10x_1 + 20x_2$

subject to,

$3x_1 + 5x_2 \leq 90$

$6x_1 + 3x_2 \leq 72$

and $x_1, x_2 \geq 0$.

Standard form:

$\text{Max} z = 10x_1 + 20x_2 + 0S_1 + 0S_2$

S.T.C

$3x_1 + 5x_2 + S_1 = 90$

$6x_1 + 3x_2 + S_2 = 72$

and $x_1, x_2, S_1, S_2 \geq 0$.

Matrix form:

$$\begin{pmatrix} 3 & 5 & 1 & 0 \\ 6 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 90 \\ 72 \end{pmatrix}$$ constant term

Starting table:

			10	20	0	0	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	90	3	5	1	0	$\frac{90}{5} = 18$
0	s_2	72	6	3	0	1	$\frac{72}{3} = 24$
$z_j - c_j$			-10	-20	0	0	

s_1 Leaves the basis

x_2 enter the basis

old eqn : 90 3 5 1 0

New eqn : $\frac{90}{5} = 18$ $\frac{3}{5}$ 1 $1/5$ 0

Change:

72	6	3	0	1
(-) 54	9/5	3	3/5	0
18	21/5	0	-3/5	1

I^{st} - Iteration:

			10	20	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
20	x_2	18	3/5	1	1/5	0
0	s_2	18	21/5	0	-3/5	1
$z_j - c_j$		360	2	0	$\frac{20}{5} = 4$	0

Here all $z_j - c_j \geq 0$.

$\therefore$ The optimum solution is

Max $z = 360$

$x_1 = 0$; $x_2 = 18$.

3. use simplex method to solve the following LPP. Max $z = 4x_1 + 7x_2$ subject to, $4x_1 + 3x_2 \leq 12$, $3x_1 + 4x_2 \leq 12$ and $x_1, x_2 \geq 0$.

soln:

canonical form:

Max $z = 4x_1 + 7x_2$

s.t.t,

$4x_1 + 3x_2 \le 12$

$3x_1 + 4x_2 \le 12$

and $x_1, x_2 \ge 0$.

Standard form:

$\text{Max } Z = 4x_1 + 7x_2 + 0S_1 + 0S_2$

S.T.C

$4x_1 + 3x_2 + S_1 = 12$

$3x_1 + 4x_2 + S_2 = 12$

and $x_1, x_2, S_1, S_2 \ge 0$.

Matrix form:

$$\begin{pmatrix} 4 & 3 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

Starting table:

			4	7	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	Ratio
0	S_1	12	4	3	1	0	$12/3 = 4$
0	S_2	12	3	4	0	1	$12/4 = 3$
$Z_j - C_j$			-4	-7 ↑	0	0	

S_2 leaves the basis

x_2 enter the basis

old eqn : 12 3 4 0 1

New eqn : $\frac{12}{4} = 3$ $\frac{3}{4}$ 1 0 $\frac{1}{4}$

Change:

	12	4	3	1	0
(-)	9	$9/4$	3	0	$3/4$
	3	$7/4$	0	1	$-3/4$

I^{st} - Iteration :

			4	7	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2
0	S_1	3	$7/4$	0	1	$-3/4$
7	x_2	3	$3/4$	1	0	$1/4$
$Z_j - C_j$		21	$5/4$	0	0	$7/4$

Here all $Z_j - C_j \ge 0$.

∴ The optimum solution is

$$\text{Max } z = 21$$

$$x_1 = 0 \; ; \; x_2 = 3.$$

4. Solve the LPP using simplex method $\text{Min } z = x_1 - 3x_2 + 2x_3$ S.T.C $3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 3x_3 \le 10$ and $x_1, x_2, x_3 \ge 0$.

Sol:

Canonical form:

$$\text{Min } z = x_1 - 3x_2 + 2x_3$$

S.T.C

$$3x_1 - x_2 + 2x_3 \le 7$$

$$-2x_1 + 4x_2 \le 12$$

$$-4x_1 + 3x_2 + 8x_3 \le 10$$

and $x_1, x_2, x_3 \ge 0$.

Simplex method MAX Z

$$\text{Min } z = -(\text{Max } z^*)$$

$$\text{Max } z^* = -\text{Min } z$$

$$= -(x_1 - 3x_2 + 2x_3)$$

$$= -x_1 + 3x_2 - 2x_3$$

Standard form:

$$\text{Max } z^* = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$$

S.T.C

$$3x_1 - x_2 + 2x_3 + s_1 = 7$$

$$-2x_1 + 4x_2 + 0x_3 + s_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$$

and

$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0.$$

Matrix form:

$$\begin{pmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & 4 & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \\ 10 \end{pmatrix}$$

Starting table:

			-1	3	-2	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	7	3	-1	2	1	0	0	$\frac{7}{-1} = -7$
0	s_2	12	-2	4	0	0	1	0	$\frac{12}{4} = 3$
0	s_3	10	-4	3	8	0	0	1	$\frac{10}{3} = 3.3$
$z_j - c_j$			1	most negative -3	2	0	0	0	

S_2 leaves and x_2 enter the basis

Change:-

	7	3	-1	2	1	0	0
(+)	-3	1/2	-1	0	0	-1/4	0
	10	5/2	0	2	1	1/4	0

	10	-4	3	8	0	0	1
(-)	9	-3/2	3	0	0	3/4	0
	1	-5/2	0	8	0	-3/4	1

			-1	3	-2	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	s_1	10	5/2	0	2	1	1/4	0	$\frac{7}{Z_1}$ $z \to 7$ $10/5/2=4$
3	x_2	3	-1/2	1	0	0	1/4	0	$\frac{12}{4}=3$ $\frac{3/2}{-1/2}=-6$
0	s_3	1	-5/2	0	8	0	-3/4	1	$\frac{10}{3}=3.3$ $-2/5$
z_j-c_j		9	-1/2	0	2	0	3/4	0	

S_1 leaves and x_1 enter the basis

Change:

	3	-1/2	1	0	0	1/4	0
(-)	-2	-1/2	0	-2/5	-1/5	-1/2	0
	5	0	1	2/5	1/5	6/20	0

	1	-5/2	0	8	0	-3/4	1
(-)	-10	-5/2	0	-2	-1	-1/4	0
	11	0	0	10	1	-2/4	1

			-1	3	-2	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
-1	x_1	4	1	0	2/5	2/5	1/10	0
3	x_2	5	0	1	2/5	1/5	3/10	0
0	x_3	11	0	0	10	1	-1/2	1
z_j-c_j		11	0	0	12/5	1/5	8/10	0

Here all $z_j - c_j \geq 0$

Max $Z = 1$

$x_1 = 4$, $x_2 = 5$ and $x_3 = 0$

Hence the optimum solns is

$$\text{Min } z = -(\text{Max } z^*)$$
$$\text{Min } z = -11$$
$$x_1 = 4,\ x_2 = 5 \text{ and } x_3 = 0.$$

5. Solve: Min $z = 8x_1 - 2x_2$ S.T.C $4x_1 + 2x_2 \le 1$, $5x_1 - 4x_2 \le 3$, and $x_1, x_2 \ge 0$.

Soln:

canonical form:

$$\text{Min } z = 8x_1 - 2x_2$$

S.T.C
$$4x_1 + 2x_2 \le 1$$
$$5x_1 - 4x_2 \le 3$$
and $x_1, x_2 \ge 0$.

$\text{Min } z^* = -(\text{Min } z^*)$

$\text{Min } z^* = -(8x_1 - 2x_2)$

$= -8x_1 + 2x_2$

Standard form:

$$\text{Min } z^* = -8x_1 + 2x_2 + 0S_1 + 0S_2.$$

S.T.C
$$4x_1 + 2x_2 + S_1 = 1$$
$$5x_1 - 4x_2 + S_2 = 3$$
and $x_1, x_2, S_1, S_2 \ge 0$.

Matrix form:

$$\begin{pmatrix} 4 & 2 & 1 & 0 \\ 5 & -4 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

Starting table: -8 2 0 0

c_B	Y_B	X_B	x_1	x_2	S_1	S_2	Ratio
0	S_1	1	4	2	1	0	1/2
0	S_1	3	5	-4	0	1	-3/4
$z_j - c_j$		0	8-0	-2	0	0	

S_1 leaves and x_2 enter the basis

old eqn : 1 4 2 1 0

New eqn : $\frac{1}{2}$ $\frac{4}{2} = 2$ $\frac{2}{2} = 1$ $\frac{1}{2}$ 0

Change:

3	5	-4	0	1
(-) -2	-8	-4	-2	0
5	+3	0	2	1

			-8	2	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
2	x_2	1/2	2	1	1/2	0
0	s_2	5	13	0	2	1
$z_j - c_j$		1	12	0	1	0

Here all $z_j - c_j \geq 0$.

$$\text{Max } z^* = 1$$

$$x_1 = 0 ; \; x_2 = 1/2$$

Hence the optimum solns is

$$\text{Min } z = -(\text{max } z^*)$$

$$\text{Min } z = -1$$

$$x_1 = 0 ; \; x_2 = 1/2$$

6. Salve : Max $z = 3x_1 + 2x_2$ S.T.C $x_1 + x_2 \leq 4$, $x_1 - x_2 \leq 2$, and $x_1, x_2 \geq 0$.

Sol:

canonical form:

$$\text{Max } z = 3x_1 + 2x_2$$

S.T.C $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and $x_1, x_2 \geq 0$.

Standard form:

$$\text{Max } z = 3x_1 + 2x_2 + 0s_1 + 0s_2$$

S.T.C $x_1 + x_2 + s_1 = 4$

$x_1 - x_2 + s_2 = 2$

and $x_1, x_2, s_1, s_2 \geq 0$.

Matrix form:

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Starting tables :-

			3	2	0	0	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio
0	s_1	4	1	1	1	0	$\frac{4}{1} = 4$
0	s_2	2	1	-1	0	1	$\frac{2}{1} = 2$
$z_j - c_j$			-3	-2	0	0	

S_2 leaves and x_1 enter the basis

			3	2	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	Ratio
0	S_1	2	0	2	1	-1	$\frac{2}{2} = 1$
3	x_1	2	1	-1	0	1	$\frac{2}{-1} = -2$
$Z_j - C_j$		6	0	-5	0	3	

Change:-

	4	1	1	1	0
(-)	2	1	-1	0	1
	2	0	2	1	-1

S_1 leaves and x_2 enter the basis

			3	2	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2
2	x_2	1	0	1	$1/2$	$-1/2$
3	x_1	3	1	0	$1/2$	$1/2$
$Z_j - C_j$		11	0	0	$5/2$	$1/2$

$2+9$ (11)

$0+3$

$\frac{2}{2}+\frac{3}{2}$ $\left(\frac{5}{2}\right)$

$-\frac{2}{2}+\frac{3}{2}$ $\left(\frac{1}{2}\right)$

Change:-

	2	1	-1	0	1
(-)	-1	0	-1	$-1/2$	$1/2$
	3	1	0	$1/2$	$1/2$

Here all $Z_j - C_j \geq 0$.

$\therefore$ The optimum solns is

Max $Z = 11$

$x_1 = 3$; $x_2 = 1$